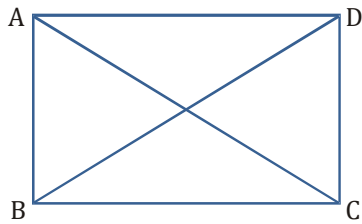


NCERT QUESTIONS WITH SOLUTIONS

EXERCISE : 8.1

1. If the diagonals of a parallelogram are equal, then show that it is a rectangle.



Sol. Given : ABCD is a parallelogram with diagonal AC = diagonal BD

To prove : ABCD is a rectangle.

Proof : In triangle ABC and ABD,

$$AB = AB \quad [\text{Common}]$$

$$AC = BD \quad [\text{Given}]$$

$$AD = BC \quad [\text{Opp. Sides of a ||gm}]$$

$$\therefore \triangle ABC \cong \triangle BAD \quad [\text{By SSS congruency}]$$

$$\Rightarrow \angle DAB = \angle CBA \quad [\text{By C.P.C.T.}] \quad \dots (i)$$

$[\because AD \parallel BC \text{ and } AB \text{ cuts them, the sum of}$

$\text{the interior angle of the same side of transversal is } 180^\circ]$

$$\angle DAB + \angle CBA = 180^\circ \quad \dots (ii)$$

$$\text{From eq. (i) and (ii), } \angle DAB = \angle CBA = 90^\circ$$

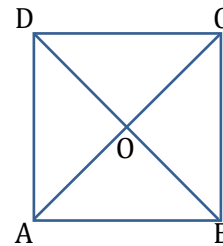
So, ABCD is a parallelogram with one of the angles equal to 90° ,

Hence, ABCD is a rectangle.

2. Show that the diagonals of a square are equal and bisect each other at right angles.

Sol. Given: ABCD is a square.

To Prove : (i) $AC = BD$ (ii) AC and BD bisect each other at right angles.



Proof: In $\triangle ABC$ and $\triangle BAD$,

$$AB = BA \quad [\text{Common}]$$

$$BC = AD \quad [\text{Opp. sides of square ABCD}]$$

$$\angle ABC = \angle BAD \quad [\text{Each} = 90^\circ (\because \text{ABCD is a square})]$$

$$\therefore \triangle ABC \cong \triangle BAD \quad [\text{SAS Rule}]$$

$$\therefore AC = BD \quad \dots (i) \quad [\text{C.P.C.T.}]$$

In $\triangle AOD$ and $\triangle BOC$

$$AD = CB \quad [\text{Opp. sides of square ABCD}]$$

$$\angle OAD = \angle OCB$$

[Alternate angles as $AD \parallel BC$ and transversal AC intersects them]

$$\angle ODA = \angle OBC$$

[Alternate angles as $AD \parallel BC$ and transversal BD intersects them]

$$\triangle AOD \cong \triangle COB \quad [\text{ASA Rule}]$$

$$\therefore OA = OC \text{ and } OB = OD \quad \dots (ii) \quad [\text{C.P.C.T.}]$$

So, O is the midpoint of AC and BD.

Now, In $\triangle AOB$ and $\triangle COB$

$$AB = BC \quad [\text{Given}]$$

$$OA = OC \quad [\text{from (ii)}]$$

$$OB = OB \quad [\text{Common}]$$

$$\therefore \triangle AOB \cong \triangle COB \quad [\text{By SSS Rule}]$$

$$\therefore \angle AOB = \angle BOC \quad [\text{C.P.C.T.}]$$

But $\angle AOB + \angle BOC = 180^\circ$ [Linear pair]

$$\angle AOB + \angle AOB = 180^\circ$$

[$\angle AOB = \angle BOC$ proved earlier]

$$\Rightarrow 2\angle AOB = 180^\circ$$

$$\Rightarrow \angle AOB = \frac{180^\circ}{2} = 90^\circ$$

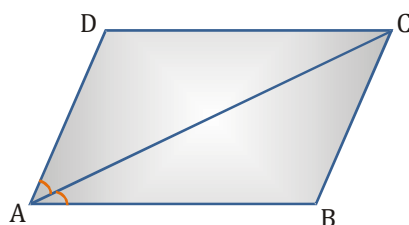
$$\therefore \angle AOB = \angle BOC = 90^\circ$$

$\therefore$ AC and BD bisect each other at right angles.

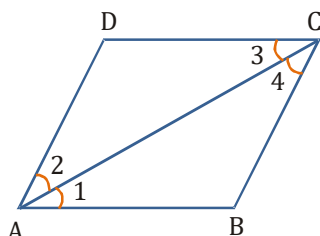
3. Diagonal AC of a parallelogram ABCD bisects $\angle A$. Show that

(i) it bisects $\angle C$ also

(ii) ABCD is a rhombus.



Sol. Given :



Diagonal AC bisects $\angle A$ of the parallelogram ABCD.

To prove :

(i) AC bisects $\angle C$

(ii) ABCD is a rhombus

Proof :

(i) Since $AB \parallel DC$ and AC intersects them.

$$\therefore \angle 1 = \angle 3 \quad [\text{Alternate angles}] \dots(i)$$

$$\text{Similarly, } \angle 2 = \angle 4 \quad \dots(ii)$$

$$\text{But } \angle 1 = \angle 2 \quad [\text{Given}] \quad \dots(iii)$$

$$\therefore \angle 3 = \angle 4$$

[Using eq. (i), (ii) and (iii)]

Thus, AC bisects $\angle C$.

(ii) since $\angle 2 = \angle 3$ (using (i) and (iii))

$$\Rightarrow AD = CD$$

[Sides opposite to equal angles]

Also, ABCD is a parallelogram.

$$\Rightarrow AD = BC \text{ and } AB = CD$$

$$\therefore AB = CD = AD = BC$$

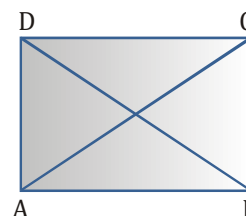
Hence, ABCD is a rhombus.

4. ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$. Show that

(i) ABCD is a square

(ii) diagonal BD bisects $\angle B$ as well as $\angle D$.

Sol. Given : ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$.



To prove :

(i) ABCD is a square

(ii) diagonal BD bisects $\angle B$ as well as $\angle D$.

Proof :

(i) $\because AB \parallel DC$ and transversal AC intersects them.

$$\text{So, } \angle BAC = \angle DCA \quad [\text{Alternate angles}]$$

$$\text{But } \angle BAC = \angle DAC \quad [\because AC \text{ bisects } \angle A]$$

$$\therefore \angle DCA = \angle DAC$$

$$\Rightarrow DA = CD$$

[Sides opposite to equal angles of a triangle are equal]

$$\text{But } AB = CD \text{ and } DA = BC$$

[Opposite side of a rectangle]

$$\therefore AB = BC = CD = DA$$

$$\text{Also, } \angle A = \angle B = \angle C = \angle D = 90^\circ$$

[$\because$ ABCD is a rectangle]

Hence, ABCD is a square.

(ii) In $\triangle BAD$ and $\triangle BCD$,

$$BA = BC \quad [\because ABCD \text{ is a square}]$$

$$AD = CD \quad [\because ABCD \text{ is a square}]$$

$$BD = BD \quad [\text{Common}]$$

$$\therefore \triangle BAD \cong \triangle BCD$$

[By SSS congruence rule]

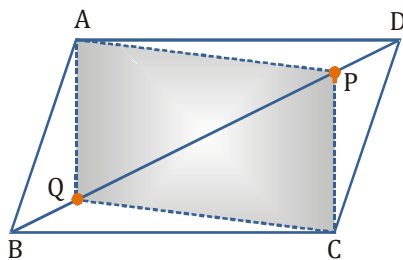
$$\therefore \angle ABD = \angle CBD \quad [\text{By C.P.C.T.}]$$

$$\angle ADB = \angle CDB \quad [\text{By C.P.C.T.}]$$

Hence, diagonal BD bisects $\angle B$ as well as $\angle D$

5. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that $DP = BQ$. Show that :

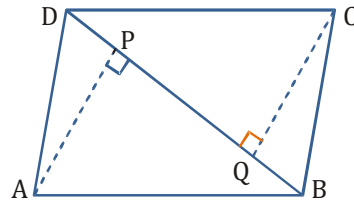
- (i) $\triangle APD \cong \triangle CQB$
- (ii) $AP = CQ$
- (iii) $\triangle AQB \cong \triangle CPD$
- (iv) $AQ = CP$
- (v) APCQ is a parallelogram



- Sol.** (i) In $\triangle APD$ and $\triangle CQB$, we have
 $DP = BQ$ [Given]
 $AD = CB$
 [Opposite sides of parallelogram ABCD]
 $\angle ADP = \angle CBQ$ [Pair of alternate angles]
 $\Rightarrow \triangle APD \cong \triangle CQB$
 [SAS congruence criteria]
 (ii) Then, by CPCT, we have $AP = CQ$
 (iii) We can prove
 $\triangle AQB \cong \triangle CPD$
 [as we have done in (i)]
 (iv) By CPCT, we have $AQ = CP$
 (v) Now, we have $AP = CQ$ and $AQ = CP$
 Hence, APCQ is a parallelogram.

6. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD. Show that

- (i) $\triangle APB \cong \triangle CQD$ (ii) $AP = CQ$



- Sol. Given :** ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD respectively.

To prove : (i) $\triangle APB \cong \triangle CQD$ (ii) $AP = CQ$

Proof :

- (i) In $\triangle APB$ and $\triangle CQD$,
 $AB = CD$ [Opposite side of $\parallel$ gm ABCD]
 $\angle ABP = \angle CDQ$
 $[\because AB \parallel DC \text{ and transversal BD intersect them}]$

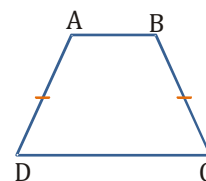
$$\angle APB = \angle CQD \quad [\text{Each} = 90^\circ]$$

$$\therefore \triangle APB \cong \triangle CQD \quad [\text{AAS Rule}]$$

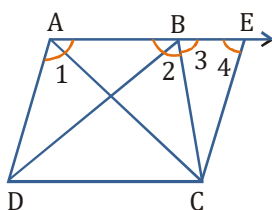
$$(ii) \quad AP = CQ \quad [\text{C.P.C.T.}]$$

7. ABCD is a trapezium in which $AB \parallel CD$ and $AD = BC$. Show that (figure)

- (i) $\angle A = \angle B$
- (ii) $\angle C = \angle D$
- (iii) $\triangle ABC \cong \triangle BAD$
- (iv) diagonal $AC =$ diagonal BD



Sol. Given :



ABCD is a trapezium.

$AB \parallel CD$ and $AD = BC$

To Prove :

(i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\triangle ABC \cong \triangle BAD$

(iv) Diagonal $AC =$ Diagonal BD

Construction : Draw $CE \parallel AD$ and extend AB to intersect CE at E .

Proof :

(i) As $AECD$ is a parallelogram.

[By construction]

$\therefore AD = EC$

But $AD = BC$ [Given]

$\therefore BC = EC$

$\Rightarrow \angle 3 = \angle 4$

[Angles opposite to equal sides are equal]

Now, $\angle 1 + \angle 4 = 180^\circ$ [Co-Interior angles]

And $\angle 2 + \angle 3 = 180^\circ$ [Linear pair]

$\Rightarrow \angle 1 + \angle 4 = \angle 2 + \angle 3$

$\Rightarrow \angle 1 = \angle 2$ [$\because \angle 3 = \angle 4$]

$\Rightarrow \angle A = \angle B$

(ii) $\angle 3 = \angle BCD$

[Alternate interior angles]

$\angle ADC = \angle 4$ [Opposite angles of a parallelogram]

But $\angle 3 = \angle 4$ [$\triangle BCE$ is an isosceles triangle]

$\therefore \angle BCD = \angle ADC$

$\therefore \angle C = \angle D$

(iii) In $\triangle ABC$ and $\triangle BAD$,

$AB = AB$ [Common]

$\angle 1 = \angle 2$ [Proved]

$AD = BC$ [Given]

$\therefore \triangle ABC \cong \triangle BAD$ [By SAS congruency]

$\Rightarrow AC = BD$ [By C.P.C.T.]

8. The angles of quadrilateral are in the ratio $3 : 5 : 9 : 13$. Find all the angles of the quadrilateral.

Sol. Let the four angles of the quadrilateral be $3x, 5x, 9x$ and $13x$.

$3x + 5x + 9x + 13x = 360^\circ$ [Sum of all the angles of quadrilateral is 360°]

$\Rightarrow 30x = 360^\circ$

$\Rightarrow x = 12^\circ$

Hence, the angles of the quadrilateral are

$3 \times 12^\circ = 36^\circ, 5 \times 12^\circ = 60^\circ,$

$9 \times 12^\circ = 108^\circ$ and $13 \times 12^\circ = 156^\circ$.

9. Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Sol. Given : ABCD is a quadrilateral where diagonals AC and BD meet at O , such that $AO = OC, OB = OD$ and $AC \perp BD$

To Prove: Quadrilateral ABCD is a rhombus,

i.e., $AB = BC = CD = DA$

Proof : In $\triangle AOB$ and $\triangle AOD$,

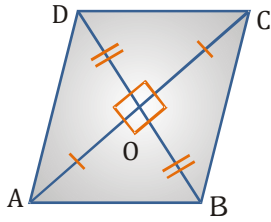
$OB = OD$ [Given]

$AO = AO$ [Common]

$\angle AOB = \angle AOD$ [Each = 90°]

$\therefore \triangle AOB \cong \triangle AOD$ [SAS Rule]

$\therefore AB = AD$ [C.P.C.T.] ... (i)



Similarly, we can prove that

$$AB = BC \quad \dots \text{(ii)}$$

$$BC = CD \quad \dots \text{(iii)}$$

$$CD = AD \quad \dots \text{(iv)}$$

From (i), (ii), (iii) and (iv), we obtain,

$$AB = BC = CD = DA$$

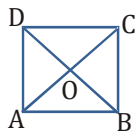
$\therefore$ Quadrilateral ABCD is a rhombus.

- 10.** Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Sol. Given : The diagonals AC and BD of a quadrilateral ABCD are equal and bisect each other at right angles.

To prove : Quadrilateral ABCD is a square.

Proof :



In $\triangle AOD$ and $\triangle BOC$,

$$OA = OC \quad [\text{Given}]$$

$$OD = OB \quad [\text{Given}]$$

$$\angle AOD = \angle COB \quad [\text{Vertically Opposite Angles}]$$

$$\therefore \triangle AOD \cong \triangle COB \quad [\text{SAS Rule}]$$

$$\therefore AD = BC \quad [\text{C.P.C.T.}]$$

$$\angle ODA = \angle OBC \quad [\text{C.P.C.T.}]$$

$$\therefore AD \parallel BC$$

$$\text{Now, } AD = CB \text{ and } AD \parallel CB$$

$$\therefore \text{Quadrilateral ABCD is a ||gm.}$$

In $\triangle AOB$ and $\triangle AOD$,

$$AO = AO \quad [\text{Common}]$$

$$OB = OD \quad [\text{Given}]$$

$$\angle AOB = \angle AOD \quad [\text{Each} = 90^\circ \text{ (Given)}]$$

$$\therefore \triangle AOB \cong \triangle AOD \quad [\text{SAS Rule}]$$

$$\therefore AB = AD$$

Now,

$$\therefore \text{ABCD is a parallelogram and } AB = AD$$

Again, in $\triangle ABC$ and $\triangle BAD$,

$$AC = BD \quad [\text{Given}]$$

$$BC = AD \quad [\because \text{ABCD is a ||gm}]$$

$$AB = BA \quad [\text{Common}]$$

$$\therefore \triangle ABC \cong \triangle BAD \quad [\text{SSS rule}]$$

$$\therefore \angle ABC = \angle BAD \quad [\text{C.P.C.T.}]$$

$$\therefore AD \parallel BC \quad [\text{Opposite sides of ||gm ABCD}]$$

and transversal AB intersects them.

$$\therefore \angle ABC + \angle BAD = 180^\circ$$

[Sum of consecutive interior angles on the same side of the transversal is 180°]

$$\therefore \angle ABC = \angle BAD = 90^\circ$$

$$\text{Similarly, } \angle BCD = \angle ADC = 90^\circ$$

$$\therefore \text{ABCD is a square.}$$

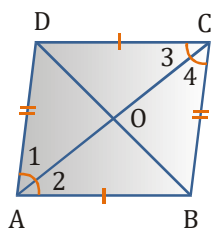
- 11.** ABCD is a rhombus. Show that diagonal AC bisects $\angle A$ as well as $\angle C$ and diagonal BD bisects $\angle B$ as well as $\angle D$.

Sol Given : ABCD is a rhombus and AC and BD are its diagonals.

To prove :

(i) Diagonal AC bisects $\angle A$ as well as $\angle C$.

(ii) Diagonal BD bisects $\angle B$ as well as $\angle D$.



Proof :

(i) In $\triangle ABC$

$AB = BC$ (sides of Rhombus)

So, $\angle 2 = \angle 4$

(Angle opposite to equal sides are equal)

But $\angle 2 = \angle 3$ (Alternate angles as $AB \parallel CD$)

So, $\angle 2 = \angle 3 = \angle 4$

But $\angle 1 = \angle 4$ (Alternate angles as $AD \parallel BC$)

So, $\angle 1 = \angle 2 = \angle 3 = \angle 4$... (1)

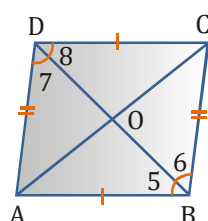
$\angle 1 = \angle 2$ by (1)

So, AC bisect $\angle A$

$\angle 3 = \angle 4$ by (1)

So, AC bisect $\angle C$

(ii) In $\triangle ABD$



$AB = AD$ (Sides of Rhombus)

So, $\angle 5 = \angle 7$

(Angle opposite to equal sides are equal)

But $\angle 7 = \angle 6$ (Alternate angles as $AD \parallel BC$)

So, $\angle 5 = \angle 6 = \angle 7$

$\angle 5 = \angle 8$ (Alternate angles as $AB \parallel CD$)

So, $\angle 5 = \angle 6 = \angle 7 = \angle 8$... (2)

$\angle 5 = \angle 6$ by (2)

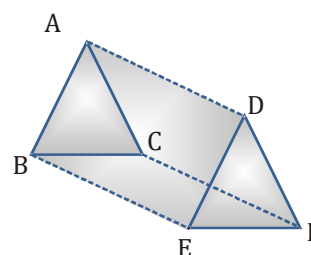
So, BD bisect $\angle B$

$\angle 7 = \angle 8$ by (2)

So, BD bisect $\angle D$

12. In $\triangle ABC$ and $\triangle DEF$, $AB = DE$, $AB \parallel DE$, $BC = EF$ and $BC \parallel EF$. Vertices A, B and C are joined to vertices D, E and F respectively. Show that :

- (i) quadrilateral ABED is a parallelogram
- (ii) quadrilateral BEFC is a parallelogram
- (iii) $AD \parallel CF$ and $AD = CF$
- (iv) quadrilateral ACFD is a parallelogram
- (v) $AC = DF$
- (vi) $\triangle ABC \cong \triangle DEF$



Sol. Given : $AB = DE$, $AB \parallel DE$, $BC = EF$ & $BC \parallel EF$

To Prove

- (i) ABED is a parallelogram.
- (ii) BEFC is a parallelogram.
- (iii) $AD \parallel CF$ and $AD = CF$
- (iv) ACFD is a parallelogram
- (v) $AC = DF$
- (vi) $\triangle ABC \cong \triangle DEF$

Proof

(i) In quad. ABED

$AB = DE$ [Given]

And $AB \parallel DE$ [Given]

$\therefore$ ABED is a parallelogram

(ii) In quad. BEFC

$BC = EF$ [Given]

And $BC \parallel EF$ [Given]

$\therefore$ BEFC is a parallelogram.

(iii) As ABED is a parallelogram.

$\therefore AD \parallel BE$ and $AD = BE$ (i)

Also, BEFC is a parallelogram

$\therefore CF \parallel BE$ and $CF = BE$ (ii)

From (i) and (ii), we get

$\therefore AD \parallel CF$ and $AD = CF$

(iv) As $AD \parallel CF$ and $AD = CF$

$\Rightarrow$ ACFD is a parallelogram.

(v) As ACFD is a parallelogram.

$\therefore AC = DF$

(vi) In $\triangle ABC$ and $\triangle DEF$,

$AB = DE$ [Given]

$BC = EF$ [Given]

$AC = DF$ [Proved]

$\therefore \triangle ABC \cong \triangle DEF$ [By SSS congruency]

EXERCISE : 8.2

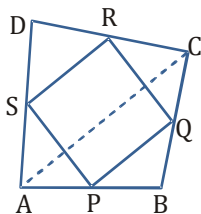
1. ABCD is a quadrilateral in which P, Q, R and S are mid points of the sides AB, BC, CD and DA (fig.) and AC is a diagonal. Show that

(i) $SR \parallel AC$ and

$$SR = \frac{1}{2} AC$$

(ii) $PQ = SR$

(iii) PQRS is a parallelogram.



Sol. Given : ABCD is a quadrilateral in which P, Q, R and S are mid-points of AB, BC, CD and DA. AC is a diagonal.

To prove :

(i) $SR \parallel AC$ and $SR = \frac{1}{2} AC$

(ii) $PQ = SR$

(iii) PQRS is a parallelogram.

Proof :

(i) In $\triangle DAC$,

$\therefore$ S is the mid-point of DA and R is the mid-point of DC

$\therefore SR \parallel AC$ and $SR = \frac{1}{2} AC$

[By Mid-point theorem]

(ii) In $\triangle BAC$,

$\therefore$ P is the mid-point of AB and Q is the mid-point of BC

$\therefore PQ \parallel AC$ and $PQ = \frac{1}{2} AC$

[By Mid-point theorem]

But from (i) $SR = \frac{1}{2} AC$ & (ii)

$$PQ = \frac{1}{2} AC$$

$\Rightarrow PQ = SR$

(iii) $PQ \parallel AC$ [From (ii)]

$SR \parallel AC$ [From (i)]

$\therefore PQ \parallel SR$

[Two lines parallel to the same line are parallel to each other]

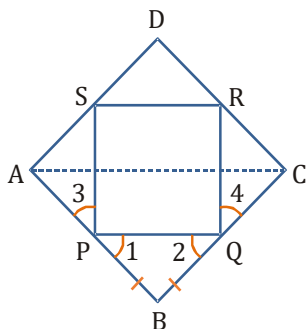
Also, $PQ = SR$ [From (ii)]

$\therefore$ PQRS is a parallelogram.

[A quadrilateral is a parallelogram if a pair of opposite sides is parallel and is of equal length]

2. ABCD is a rhombus and P, Q, R and S are the mid points of sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

Sol.



Given : P, Q, R and S are the mid-points of respective sides AB, BC, CD and DA of rhombus. PQ, QR, RS and SP are joined.

To prove : PQRS is a rectangle.

Construction : Join A and C.

Proof : In $\triangle ABC$, P is the mid-point of AB and Q is the mid-point of BC.

$$\therefore PQ \parallel AC \text{ and } PQ = \frac{1}{2}AC \quad \dots(i)$$

(By midpoint theorem)

In $\triangle ADC$, R is the mid-point of CD and S is the mid-point of AD.

$$\therefore SR \parallel AC \text{ and } SR = \frac{1}{2}AC \quad \dots(ii)$$

(By midpoint theorem)

From eq. (i) and (ii), $PQ \parallel SR$ and $PQ = SR$

$\therefore$ PQRS is a parallelogram.

Now ABCD is a rhombus [Given]

$$\therefore AB = BC$$

$$\Rightarrow \frac{1}{2}AB = \frac{1}{2}BC \Rightarrow PB = BQ$$

$$\therefore \angle 1 = \angle 2$$

[Angles opposite to equal sides are equal]

Now in triangles APS and CQR, we have,
 $AP = CQ$

[P and Q are the mid-points of AB and BC and $AB = BC$]

Similarly, $AS = CR$ and $PS = QR$

[Opposite sides of a parallelogram]

$$\therefore \triangle APS \cong \triangle CQR \text{ [By SSS congruency]}$$

$$\Rightarrow \angle 3 = \angle 4 \quad \text{[By C.P.C.T.]}$$

Now, we have $\angle 1 + \angle SPQ + \angle 3 = 180^\circ$

And $\angle 2 + \angle PQR + \angle 4 = 180^\circ$

$$\therefore \angle 1 + \angle SPQ + \angle 3 = \angle 2 + \angle PQR + \angle 4$$

Since $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ [Proved above]

$$\therefore \angle SPQ = \angle PQR \quad \dots(iii)$$

Now PQRS is a parallelogram [Proved above]

$$\therefore \angle SPQ + \angle PQR = 180^\circ \quad \dots(iv)$$

[Co-Interior angles]

Using eq. (iii) and (iv),

$$\angle SPQ + \angle SPQ = 180^\circ$$

$$\Rightarrow 2\angle SPQ = 180^\circ$$

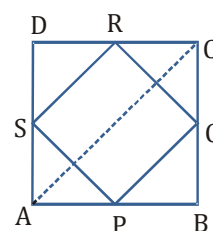
$$\Rightarrow \angle SPQ = 90^\circ$$

Hence, PQRS is a rectangle.

3. ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Sol. Given : A rectangle ABCD in which P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. PQ, QR, RS and SP are joined.

To prove : PQRS is a rhombus.



Construction : Join AC.

Proof : In $\triangle ABC$, P and Q are the mid-points of sides AB, BC respectively.

$$\therefore PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \quad \dots(i)$$

[Mid- Point Theorem]

In $\triangle ADC$, R and S are the mid-points of sides CD, AD respectively.

$$\therefore SR \parallel AC \text{ and } SR = \frac{1}{2} AC \quad \dots(ii)$$

[Mid- Point Theorem]

From eq.(i) and (ii), $PQ \parallel SR$ and

$$PQ = SR \quad \dots(iii)$$

$\therefore PQRS$ is a parallelogram.

Now ABCD is a rectangle. [Given]

$$\therefore AD = BC$$

$$\Rightarrow \frac{1}{2} AD = \frac{1}{2} BC$$

$$\Rightarrow AS = BQ \quad \dots(iv)$$

In triangles APS and BPQ,

$$AP = BP \quad [P \text{ is the mid-point of } AB]$$

$$\angle PAS = \angle PBQ \quad [\text{Each } 90^\circ]$$

$$\text{And } AS = BQ \quad [\text{From eq. (iv)}]$$

$$\therefore \triangle APS \cong \triangle BPQ$$

[By SAS congruency]

$$\Rightarrow PS = PQ \quad [\text{By C.P.C.T.}] \quad \dots(v)$$

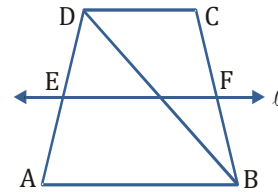
From eq.(iii) and (v), we get that PQRS is a parallelogram.

$$\Rightarrow PS = PQ$$

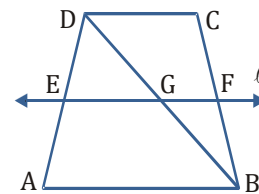
$\Rightarrow$ Two adjacent sides are equal.

Hence, PQRS is a rhombus.

4. ABCD is a trapezium in which $AB \parallel DC$, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (fig.). Show that F is the mid-point of BC.



Sol. Line $\ell \parallel AB$ and passes through E.



Line ℓ meets BC at F and BD at G.

In $\triangle ABD$, E is mid-point of AD and $EG \parallel AB$.

$\Rightarrow$ G is mid-point of BD.

[Converse of Mid Point Theorem]

Also, $\ell \parallel AB$ and $AB \parallel CD$

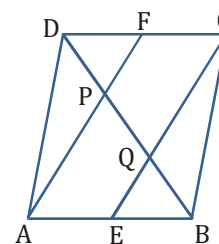
$\Rightarrow \ell \parallel CD$

$\Rightarrow$ F is mid-point of BC.

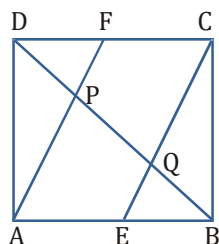
[$\because$ G is mid-point of BD]

[Converse of Mid Point Theorem]

5. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (fig.). Show that the line segments AF and EC trisect the diagonal BD.



Sol. Given : ABCD is a parallelogram. E and F are midpoints of AB and DC respectively.



To prove : $DP = PQ = QB$

Proof : Since E and F are the mid-points of AB and CD respectively.

$$\therefore AE = \frac{1}{2}AB \text{ and } CF = \frac{1}{2}CD \quad \dots(i)$$

But ABCD is a parallelogram.

$$\therefore AB = CD \text{ and } AB \parallel DC$$

$$\Rightarrow \frac{1}{2}AB = \frac{1}{2}CD \text{ and } AB \parallel DC$$

$$\Rightarrow AE = FC \text{ and } AE \parallel FC \quad [\text{From eq. (i)}]$$

$\therefore$ AECF is a parallelogram.

$$\Rightarrow FA \parallel CE$$

$$\Rightarrow FP \parallel CQ$$

[FP is a part of FA and CQ is a part of CE] ... (ii)

Since the line segment drawn through the mid-point of one side of a triangle and parallel to the other side bisects the third side.

In $\triangle DCQ$, F is the mid-point of CD and

$$\Rightarrow FP \parallel CQ$$

$\therefore$ P is the mid-point of DQ.

$$\Rightarrow DP = PQ \quad \dots (iii)$$

Similarly, In $\triangle ABP$, E is the mid-point of AB and

$$\Rightarrow EQ \parallel AP$$

$\therefore$ Q is the mid-point of BP.

$$\Rightarrow BQ = PQ \quad \dots (iv)$$

From eq.(iii) and (iv),

$$DP = PQ = BQ \quad \dots (v)$$

Now,

$$BD = BQ + PQ + DP = BQ + BQ + BQ = 3BQ$$

$$\Rightarrow BQ = \frac{1}{3}BD \quad \dots (vi)$$

From eq (v) and (vi),

$$DP = PQ = BQ = \frac{1}{3}BD$$

$\Rightarrow$ Points P and Q trisect BD. So, AF and CE trisect BD.

Hence, AF and CE trisect the diagonal BO.

6. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

(i) D is the mid-point of AC

(ii) $MD \perp AC$

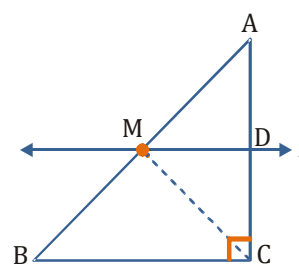
(iii) $CM = MA = \frac{1}{2}AB$

Sol. (i) Through M, we draw line $\ell \parallel BC$. ℓ intersects AC at D.

M is a midpoint of AB

$\Rightarrow$ D is mid-point of AC.

[By converse of mid-point theorem]



(ii) $\angle ADM = \angle ACB = 90^\circ$

[Corresponding angles]

$\Rightarrow \angle ADM = 90^\circ$

$\Rightarrow MD \perp AC$.

(iii) In $\triangle CMD$ and $\triangle AMD$;

$$CD = AD, MD = MD$$

$$\text{and } \angle CDM = \angle ADM$$

$$[\text{Each} = 90^\circ]$$

$$\text{Therefore, } \triangle CMD \cong \triangle AMD$$

$$[\text{SAS congruence rule}]$$

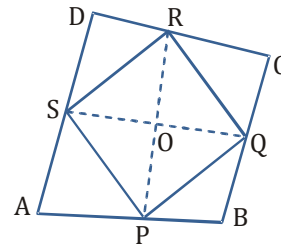
$$\Rightarrow CM = AM; \text{ Also } AM = \frac{1}{2}AB$$

$$\text{So, } CM = \frac{1}{2}AB$$

$$AM = CM = \frac{1}{2}AB$$

7. Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Sol. P, Q, R and S are the mid-points of the sides AB, BC, CD and AD of the quadrilateral ABCD.



We have to prove that, PR and QS bisect each other. Now, join PQ, QR, RS and PS.

Here, we can prove that PQRS is a parallelogram (as in solution 1).

Now, PR and QS are the diagonals of the parallelogram PQRS.

$\therefore$ PR and QS bisect each other as diagonals of parallelogram bisect each other.