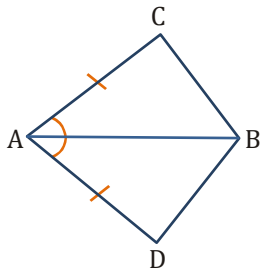


NCERT QUESTIONS WITH SOLUTIONS

EXERCISE : 7.1

1. In quadrilateral ACBD, $AC = AD$ and AB bisects $\angle A$. Show that $\triangle ABC \cong \triangle ABD$. What can you say about BC and BD ?



Sol. Given : In quadrilateral ACBD, $AC = AD$ and AB bisect $\angle A$.

To prove : $\triangle ABC \cong \triangle ABD$

Proof : In $\triangle ABC$ and $\triangle ABD$

$AC = AD$ (Given)

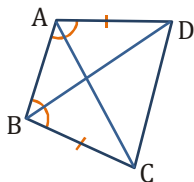
$AB = AB$ (Common)

$\angle CAB = \angle DAB$ (AB bisect $\angle A$)

$\therefore \triangle ABC \cong \triangle ABD$ (by SAS criteria)

$BC = BD$ (by CPCT)

2. ABCD is a quadrilateral in which $AD = BC$ and $\angle DAB = \angle CBA$. Prove that



(i) $\triangle ABD \cong \triangle BAC$

(ii) $BD = AC$

(iii) $\angle ABD = \angle BAC$

Sol. In $\triangle ABD$ and $\triangle BAC$,

$AD = BC$ (Given)

$\angle DAB = \angle CBA$ (Given)

$AB = AB$ (Common side)

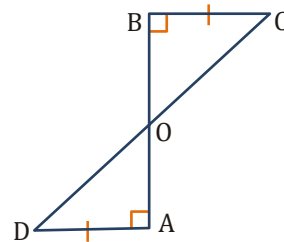
$\therefore$ By SAS congruence rule, we have

$\triangle ABD \cong \triangle BAC$

Also, by CPCT, we have

$BD = AC$ and $\angle ABD = \angle BAC$

3. AD and BC are equal perpendiculars to a line segment AB . Show that CD bisects AB .



Sol. Given :

AD and BC are equal perpendiculars to line AB .

To prove : CD bisects AB

Proof : In $\triangle OAD$ and $\triangle OBC$

$AD = BC$ (Given)

$\angle OAD = \angle OBC$ (Each 90°)

$\angle AOD = \angle BOC$

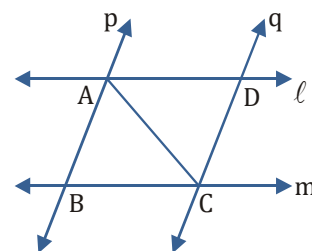
(Vertically opposite angles)

$\triangle OAD \cong \triangle OBC$ (AAS rule)

$OA = OB$ (by CPCT)

$\therefore CD$ bisects AB

4. ℓ and m are two parallel lines intersected by another pair of parallel lines p and q . Show that $\triangle ABC \cong \triangle CDA$.



Sol. In $\triangle ABC$ and $\triangle CDA$

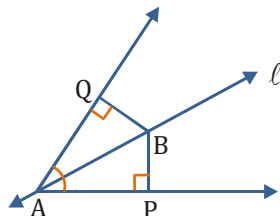
$\angle CAB = \angle ACD$ (Pair of alternate angle)

$\angle BCA = \angle DAC$ (Pair of alternate angle)

$AC = AC$ (Common side)

$\therefore \triangle ABC \cong \triangle CDA$ (ASA criteria)

5. Line ℓ is the bisector of an angle $\angle A$ and B is any point on ℓ . BP and BQ are perpendiculars from B to the arms of $\angle A$. Show that :



- (i) $\triangle APB \cong \triangle AQB$
 (ii) $BP = BQ$ or B is equidistant from the arms of $\angle A$

Sol. Given : line ℓ is bisector of angle A and B is any point on ℓ . BP and BQ are perpendicular from B to arms of $\angle A$

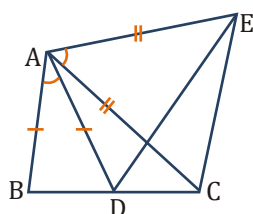
To prove :

- (i) $\triangle APB \cong \triangle AQB$
 (ii) $BP = BQ$

Proof :

- (i) In $\triangle APB$ and $\triangle AQB$
 $\angle BAP = \angle BAQ$ (ℓ is bisector)
 $AB = AB$ (common)
 $\angle BPA = \angle BQA$ (Each 90°)
 $\therefore \triangle APB \cong \triangle AQB$ (AAS rule)
 (ii) $\triangle APB \cong \triangle AQB$
 $BP = BQ$ (By CPCT)

6. In figure, $AC = AE$, $AB = AD$ and $\angle BAD = \angle EAC$. Show that $BC = DE$.



Sol. Given : $AC = AE$
 $AB = AD$,
 $\angle BAD = \angle EAC$

To prove : $BC = DE$

Proof : In $\triangle ABC$ and $\triangle ADE$

$$AB = AD \quad (\text{Given})$$

$$AC = AE \quad (\text{Given})$$

$$\angle BAD = \angle EAC$$

Add $\angle DAC$ to both

$$\Rightarrow \angle BAD + \angle DAC = \angle DAC + \angle EAC$$

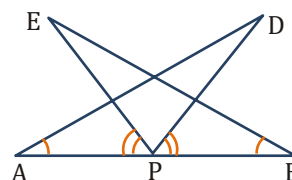
$$\angle BAC = \angle DAE$$

$$\triangle ABC \cong \triangle ADE \quad (\text{SAS rule})$$

$$BC = DE \quad (\text{By CPCT})$$

7. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$. Show that

- (i) $\triangle DAP \cong \triangle EBP$
 (ii) $AD = BE$



Sol. $\angle EPA = \angle DPB$ (Given)

$$\Rightarrow \angle EPA + \angle DPE = \angle DPB + \angle DPE$$

$$\Rightarrow \angle APD = \angle BPE \quad \dots(1)$$

Now, in $\triangle DAP$ and $\triangle EBP$, we have

$$AP = PB \quad (\because P \text{ is mid point of } AB)$$

$$\angle PAD = \angle PBE$$

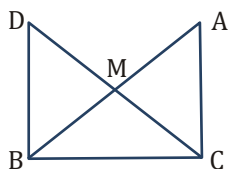
$$\left\{ \begin{array}{l} \because \angle PAD = \angle BAD, \angle PBE = \angle ABE \\ \text{and we are given that } \angle BAD = \angle ABE \end{array} \right\}$$

$$\text{Also, } \angle APD = \angle BPE \quad (\text{By 1})$$

$$\therefore \triangle DAP \cong \triangle EBP (\text{By ASA congruence})$$

$$\Rightarrow AD = BE \quad (\text{By CPCT})$$

8. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B. Show that :



- (i) $\triangle AMC \cong \triangle BMD$
 (ii) $\angle DBC$ is a right angle.
 (iii) $\triangle DBC \cong \triangle ACB$
 (iv) $CM = \frac{1}{2}AB$

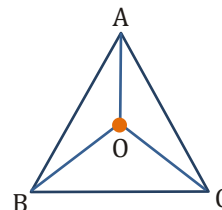
Sol. (i) In $\triangle AMC \cong \triangle BMD$,
 $AM = BM$ ($\because$ M is mid point of AB)
 $\angle AMC = \angle BMD$
 (Vertically opposite angles)
 $CM = DM$ (Given)
 $\therefore \triangle AMC \cong \triangle BMD$ (By SAS congruence)
 (ii) $\angle AMC = \angle BMD$,
 $\Rightarrow \angle ACM = \angle BDM$ (By CPCT)
 $\Rightarrow CA \parallel BD$
 $\Rightarrow \angle BCA + \angle DBC = 180^\circ$
 $\Rightarrow \angle DBC = 90^\circ$ ($\because \angle BCA = 90^\circ$)
 (iii) In $\triangle DBC$ and $\triangle ACB$,
 $DB = AC$ ($\because \triangle BMD \cong \triangle AMC$)
 $\angle DBC = \angle ACB$ (Each = 90°)
 $BC = CB$ (Common side)
 $\therefore \triangle DBC \cong \triangle ACB$ (By SAS congruence)

- (iv) In $\triangle DBC \cong \triangle ACB$
 $\Rightarrow CD = AB$... (1)
 $\Rightarrow CM = DM$ (given)
 $\Rightarrow CM = DM = \frac{1}{2}CD$
 $\Rightarrow CD = 2CM$... (2)
 From (1) and (2),
 $2CM = AB$
 $\Rightarrow CM = \frac{1}{2}AB$

EXERCISE : 7.2

1. In an isosceles triangle ABC, with $AB = AC$, the bisectors of $\angle B$ and $\angle C$ intersect each other at O. Join A to O. Show that :
 (i) $OB = OC$ (ii) AO bisects $\angle A$.

Sol.



- (i) In $\triangle ABC$, OB and OC are bisectors of $\angle B$ and $\angle C$.
 $\therefore \angle OBC = \frac{1}{2}\angle B$... (1)
 $\angle OCB = \frac{1}{2}\angle C$... (2)
 Also, $AB = AC$ (Given)
 $\Rightarrow \angle B = \angle C$... (3)
 From (1), (2), (3), we have $\angle OBC = \angle OCB$
 Now, in $\triangle OBC$, we have $\angle OBC = \angle OCB$
 $\Rightarrow OB = OC$
 (Sides opposite to equal angles are equal)

$$(ii) \quad \angle OBA = \frac{1}{2} \angle B \text{ and } \angle OCA = \frac{1}{2} \angle C$$

$$\Rightarrow \angle OBA = \angle OCA \quad (\because \angle B = \angle C)$$

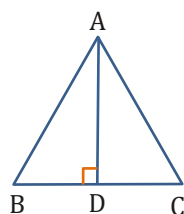
$$AB = AC \text{ and } OB = OC$$

$$\therefore \triangle OAB \cong \triangle OAC \text{ (SAS congruence criteria)}$$

$$\Rightarrow \angle OAB = \angle OAC$$

$$\Rightarrow AO \text{ bisects } \angle A.$$

2. In $\triangle ABC$, AD is the perpendicular bisector of BC. Show that $\triangle ABC$ is an isosceles triangle in which $AB = AC$.



Sol. Given : In $\triangle ABC$, AD is perpendicular bisector of BC.

To Prove : $\triangle ABC$ is isosceles $\triangle$ with $AB = AC$

Proof : In $\triangle ADB$ and $\triangle ADC$

$$\angle ADB = \angle ADC \quad (\text{Each } 90^\circ)$$

$$DB = DC \quad (\text{AD is } \perp \text{ bisector of BC})$$

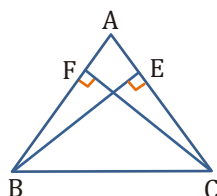
$$AD = AD \quad (\text{Common})$$

$$\triangle ADB \cong \triangle ADC \quad (\text{By SAS rule})$$

$$AB = AC \quad (\text{By CPCT})$$

$$\therefore \triangle ABC \text{ is an isosceles } \triangle \text{ with } AB = AC$$

3. ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively. Show that these altitudes are equal.



Sol. In $\triangle ABE$ and $\triangle ACF$, we have

$$\angle BEA = \angle CFA \quad (\text{Each} = 90^\circ)$$

$$\angle A = \angle A \quad (\text{Common angle})$$

$$AB = AC \quad (\text{Given})$$

$$\therefore \triangle ABE \cong \triangle ACF$$

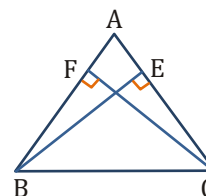
$$(\text{By AAS congruence criteria})$$

$$\Rightarrow BE = CF \quad (\text{By CPCT})$$

4. ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (See figure). Show that

$$(i) \quad \triangle ABE \cong \triangle ACF$$

$$(ii) \quad AB = AC, \text{ i.e., } \triangle ABC \text{ is an isosceles triangle.}$$



Sol. (i) In $\triangle ABE$ and $\triangle ACF$,

$$\text{we have } \angle A = \angle A \quad (\text{Common})$$

$$\angle AEB = \angle AFC \quad (\text{Each} = 90^\circ)$$

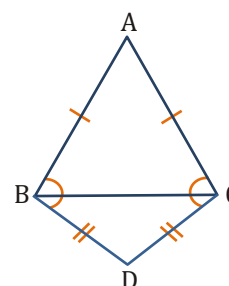
$$BE = CF \quad (\text{Given})$$

$$\therefore \triangle ABE \cong \triangle ACF \quad (\text{By AAS congruence})$$

$$(ii) \quad \triangle ABE \cong \triangle ACF$$

$$\Rightarrow AB = AC \quad (\text{By CPCT})$$

5. ABC and DCB are two isosceles triangles on the same base BC (see figure). Show that $\angle ABD = \angle ACD$.



Sol. Given : ABC and BCD are two isosceles triangle on common base BC.

To prove : $\angle ABC = \angle ACD$

Proof : ABC is an isosceles

Triangle on base BC

$$\therefore \angle ABC = \angle ACB \quad \dots (1)$$

$\therefore$ DBC is an isosceles Δ on base BC.

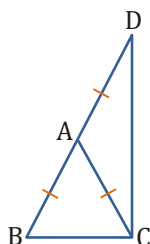
$$\angle DBC = \angle DCB \quad \dots (2)$$

Adding (1) and (2)

$$\angle ABC + \angle DBC = \angle ACB + \angle DCB$$

$$\Rightarrow \angle ABD = \angle ACD$$

6. ΔABC is an isosceles triangle in which $AB = AC$. Side BA is produced to D such that $AD = AB$ (see figure). Show that $\angle BCD$ is a right angle.



Sol. In ΔABC , $AB = AC$

$$\Rightarrow \angle ACB = \angle ABC \quad \dots (1)$$

In ΔACD ,

$$AD = AB \quad (\text{By construction})$$

$$\Rightarrow AD = AC$$

$$\Rightarrow \angle ACD = \angle ADC \quad \dots (2)$$

Adding (1) and (2),

$$\angle ACB + \angle ACD = \angle ABC + \angle ADC$$

$$\Rightarrow \angle BCD = \angle ABC + \angle ADC$$

In ΔBDC ,

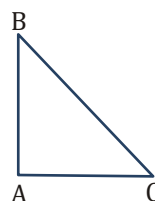
$$\angle ABC + \angle BCD + \angle CDB = 180^\circ$$

$$\Rightarrow 2 \angle BCD = 180^\circ$$

$$\Rightarrow \angle BCD = 90^\circ$$

7. ABC is a right angled triangle in which $\angle A = 90^\circ$ and $AB = AC$. Find $\angle B$ and $\angle C$.

Sol.



In ΔABC

$$AB = AC$$

$$\angle B = \angle C \quad \dots (1)$$

(angles opposite to equal sides are equal)

In ΔABC

$$\angle A + \angle B + \angle C = 180^\circ$$

$$90^\circ + \angle B + \angle C = 180^\circ$$

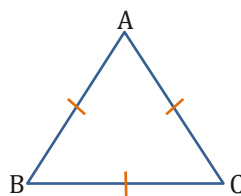
$$\angle B + \angle C = 90^\circ \quad \dots (2)$$

from (1) and (2)

$$\angle B = \angle C = 45^\circ$$

8. Show that the angles of an equilateral triangle are 60° each.

Sol.



ΔABC is equilateral triangle.

$$\Rightarrow AB = BC = CA$$

Now, $AB = BC$

$$\Rightarrow BA = BC$$

$$\Rightarrow \angle C = \angle A \quad \dots (1)$$

$$\text{Similarly, } \angle A = \angle B \quad \dots (2)$$

From (1) and (2),

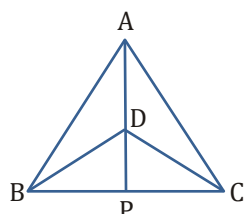
$$\angle A = \angle B = \angle C \quad \dots (3)$$

$$\text{Also, } \angle A + \angle B + \angle C = 180^\circ \quad \dots (4)$$

$$\Rightarrow \angle A = \angle B = \angle C = \frac{1}{3} \times 180^\circ = 60^\circ$$

EXERCISE : 7.3

1. $\triangle ABC$ and $\triangle DBC$ are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see figure). If AD is extended to intersect BC at P , show that



- (i) $\triangle ABD \cong \triangle ACD$
 (ii) $\triangle ABP \cong \triangle ACP$
 (iii) AP bisects $\angle A$ as well as $\angle D$
 (iv) AP is the perpendicular bisector of BC .

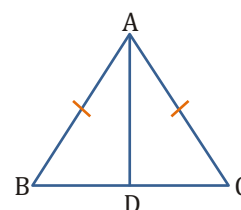
Sol. (i) In $\triangle ABD$ and $\triangle ACD$,
 $AB = AC$ ($\because \triangle ABC$ is isosceles)
 $DB = DC$ ($\because \triangle DBC$ is isosceles)
 $AD = AD$ (Common side)
 $\therefore \triangle ABD \cong \triangle ACD$ (By SSS congruence rule)
 (ii) Now, $\triangle ABD \cong \triangle ACD$
 $\Rightarrow \angle BAD = \angle CAD$ (By CPCT) ... (1)
 In $\triangle ABP$ and $\triangle ACP$,
 $AB = AC$ ($\because \triangle ABC$ is isosceles)
 $\Rightarrow \angle BAP = \angle CAP$ (By 1)
 $AP = AP$ (common side)
 $\therefore \triangle ABP \cong \triangle ACP$ (By SAS congruence rule)
 (iii) $\triangle ABD \cong \triangle ACD$ (Proved above)
 $\angle BAD = \angle CAD$ (by CPCT)
 $\angle ADB = \angle ADC$ (by CPCT)
 $180 - \angle ADB = 180 - \angle ADC$
 $\Rightarrow \angle BDP = \angle CDP$
 AP bisects $\angle A$ as well as $\angle D$

- (iv) $\triangle ABP \cong \triangle ACP$ (Proved above)
 $\Rightarrow BP = CP$ (By CPCT)
 $\Rightarrow AP$ bisects BC
 $\angle APB = \angle APC$ (By CPCT)
 $\angle APB + \angle APC = 180^\circ$
 $2\angle APB = 180^\circ$
 $\angle APB = 90^\circ$
 AP is perpendicular bisector of BC

2. AD is an altitude of an isosceles triangle ABC in which $AB = AC$. Show that

- (i) AD bisects BC
 (ii) AD bisects $\angle A$

Sol. Given : AD is an altitude of an isosceles triangle ABC in which $AB = AC$.



To Prove :

- (i) AD bisect BC .
 (ii) AD bisect $\angle A$.

Proof : (i) In right $\triangle ADB$ and right $\triangle ADC$.

Hyp. $AB = AC$

$\angle ADB = \angle ADC$ (Each 90°)

Side $AD = AD$ (Common)

$\triangle ADB \cong \triangle ADC$ (RHS rule)

$\Rightarrow BD = CD$ (By CPCT)

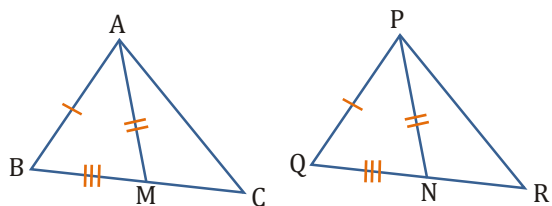
$\Rightarrow AD$ bisect BC

(ii) $\triangle ADB \cong \triangle ADC$

$\angle BAD = \angle CAD$ (By CPCT)

$\Rightarrow AD$ bisect $\angle A$

3. Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of $\triangle PQR$ (see figure). Show that :



(i) $\Delta ABM \cong \Delta PQN$

(ii) $\Delta ABC \cong \Delta PQR$

Sol. (i) $BM = \frac{1}{2} BC$ ($\because$ M is mid-point of BC)

$$QN = \frac{1}{2} QR \quad (\because N \text{ is mid-point of } QR)$$

$\Rightarrow BM = QN$ ($\because BC = QR$ is given)

Now, in ΔABM and ΔPQN , we have

$AB = PQ$ (Given)

$BM = QN$ (Proved)

$AM = PN$ (Given)

$\therefore \Delta ABM \cong \Delta PQN$ (SSS congruence criteria)

(ii) $\Delta ABM \cong \Delta PQN$

$\Rightarrow \angle ABM = \angle PQN$

$\angle B = \angle Q$ (By CPCT)

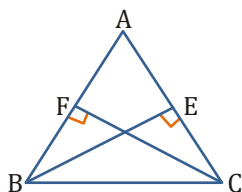
Now, in ΔABC and ΔPQR ,

$AB = PQ, \angle B = \angle Q$ and $BC = QR$

$\Rightarrow \Delta ABC \cong \Delta PQR$ [by SAS congruence]

4. BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

Sol. Given BE and CF are two altitude of ΔABC .



To prove : ΔABC is isosceles.

Proof : In right ΔBEC and right ΔCFB

side $BE =$ side CF (Given)

Hyp. $BC =$ Hyp CB (Common)

$\angle BEC = \angle BFC$ (Each 90°)

$\Delta BEC \cong \Delta CFB$ (RHS Rule)

$\therefore \angle BCE = \angle CBF$ (By CPCT)

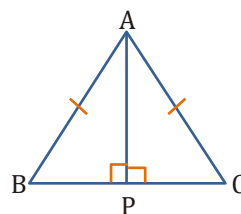
$AB = AC$

(Side opp. to equal angles are equal)

ΔABC is isosceles.

5. ABC is an isosceles triangle with $AB = AC$. Draw $AP \perp BC$ to show that $\angle B = \angle C$

Sol.



In ΔAPB and ΔAPC

$AB = AC$ (Given)

$\angle APB = \angle APC$ (Each $= 90^\circ$)

$AP = AP$ (common side)

Therefore, by RHS congruence criteria, we have

$\Delta APB \cong \Delta APC$

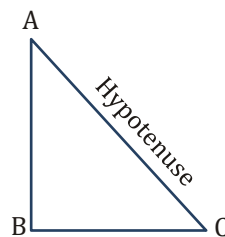
$\Rightarrow \angle ABP = \angle ACP$ (By CPCT)

$\Rightarrow \angle B = \angle C$

EXERCISE : 7.4

1. Show that in a right angled triangle, the hypotenuse is the longest side.

Sol.



ΔABC is right angled at B.

AC is hypotenuse.

Now, $\angle B = 90^\circ$

and $\angle A + \angle C = 90^\circ$

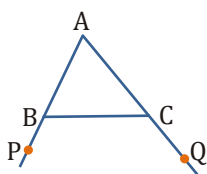
$\Rightarrow \angle A < 90^\circ$ and $\angle C < 90^\circ$

$\Rightarrow \angle B > \angle A$ and $\angle B > \angle C$

$\Rightarrow AC > BC$ and $AC > AB$.

$\therefore$ Hypotenuse AC is the longest side of the right angled $\triangle ABC$.

2. In figure, sides AB and AC of $\triangle ABC$ are extended to points P and Q respectively. Also, $\angle PBC < \angle QCB$. Show that $AC > AB$



Sol. Sides AB and AC of $\triangle ABC$ are extended to points P and Q

To prove : $AC > AB$

Proof : $\angle PBC < \angle QCB$ (Given)

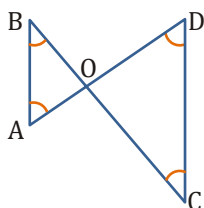
$180^\circ - \angle PBC > 180^\circ - \angle QCB$

$\angle ABC > \angle ACB$

$\Rightarrow AC > AB$

(sides opposite to greater angle is longer)

3. In fig, $\angle B < \angle A$ and $\angle C < \angle D$. Show that $AD < BC$.



Sol. $\angle B < \angle A$ in $\triangle OAB$

$\Rightarrow OA < OB$... (1)

Also, $\angle C < \angle D$ in $\triangle OCD$

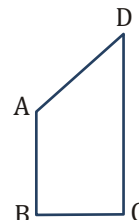
$\Rightarrow OD < OC$... (2)

Adding (1) and (2),

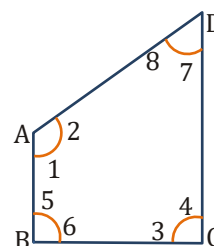
$OA + OD < OB + OC$

$\Rightarrow AD < BC$

4. AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (see figure). Show that $\angle A > \angle C$ and $\angle B > \angle D$.



Sol.



In quadrilateral ABCD, AB is the smallest side and CD is the longest side. Join AC and BD.

In $\triangle ABC$,

$BC > AB$ ($\because$ AB is smallest side)

$\Rightarrow \angle 1 > \angle 3$... (1)

In $\triangle ACD$,

$CD > AD$ ($\because$ CD is longest side)

$\Rightarrow \angle 2 > \angle 4$... (2)

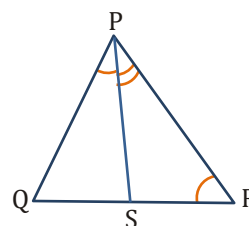
Adding (1) and (2), we have

$\angle 1 + \angle 2 > \angle 3 + \angle 4$

$\Rightarrow \angle A > \angle C$

Similarly, we can prove that $\angle B > \angle D$

5. In figure, $PR > PQ$ and PS bisects $\angle QPR$. Prove that $\angle PSR > \angle PSQ$.



Sol. Given : $PR > PQ$, PS bisect $\angle QPR$.

To prove : $\angle PSR > \angle PSQ$

Proof : In $\triangle PQR$

$PR > PQ$ (Given)

$\angle PQR > \angle PRQ$

(angle opposite to longer side is greater)

and $\angle PQS > \angle PRS$... (1)

PS bisects $\angle QPR$

$\Rightarrow \angle QPS = \angle RPS$... (2)

From (1) and (2)

$\angle PQS + \angle QPS > \angle PRS + \angle RPS$

$\angle PSR > \angle PSQ$

(by exterior angle property)

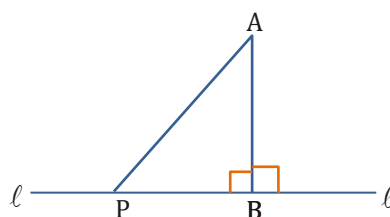
Hence proved.

6. Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

Sol. Let us have AB as the perpendicular line segment and AP is any other line segment. Now $\triangle ABP$ is right angled and AP is hypotenuse.

Here, $\angle B > \angle P$ ($\because \angle B = 90^\circ$)

$\Rightarrow AP > AB$



Thus, perpendicular line segment is smallest.