

NCERT QUESTIONS WITH SOLUTIONS

FIGURE IT OUT-01

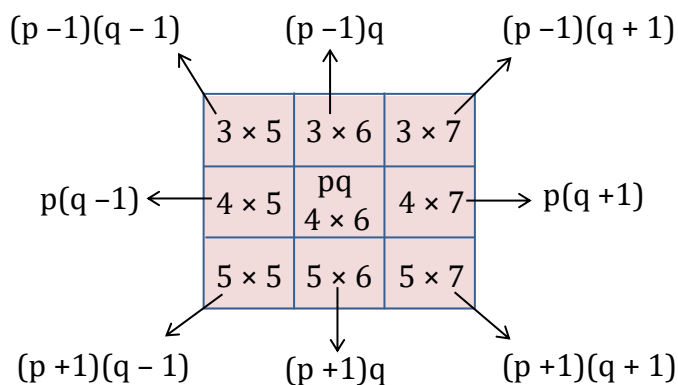
1. Observe the multiplication grid below. Each number inside the grid is formed by multiplying two numbers. If the middle number of a 3×3 frame is given by the expression pq , as shown in the figure, write the expressions for the other numbers in the grid.

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

3×5	3×6	3×7
4×5	4×6	4×7
5×5	5×6	5×7

	pq	

Sol.



2. Expand the following products.

(i) $(3 + u)(v - 3)$ (ii) $\frac{2}{3}(15 + 6a)$ (iii) $(10a + b)(10c + d)$

(iv) $(3 - x)(x - 6)$ (v) $(-5a + b)(c + d)$ (vi) $(5 + z)(y + 9)$

Sol. (i) we have, $(3 + u)(v - 3)$

$$= 3(v - 3) + u(v - 3)$$

$$= 3v - 9 + uv - 3u$$

$$= 3v - 3u + uv - 9$$

(ii) Here, $\frac{2}{3}(15 + 6a)$

$$= \frac{2}{3} \times 15 + \frac{2}{3} \times 6a$$

$$= 10 + 4a$$

(iii) Here, $(10a + b)(10c + d)$

$$= 10a \times 10c + 10a \times d + b \times 10c + b \times d$$

$$= 100ac + 10ad + 10bc + bd$$

(iv) Here, $(3 - x)(x - 6)$

$$= 3(x - 6) - x(x - 6)$$

$$= 3x - 18 - x^2 + 6x$$

$$= -x^2 + 9x - 18$$

(v) we have, $(-5a + b)(c + d)$

$$= -5a(c + d) + b(c + d)$$

$$= -5ac - 5ad + bc + bd$$

(vi) we have, $(5 + z)(y + 9)$

$$= 5(y + 9) + z(y + 9)$$

$$= 5y + 45 + yz + 9z$$

3. Find 3 examples where the product of two numbers remains unchanged when one of them is increased by 2 and the other is decreased by 4.

Sol. Let the number be a and b

$$\text{Then, } ab = (a + 2)(b - 4)$$

$$\Rightarrow ab = ab - 4a + 2b - 8$$

$$\Rightarrow ab - ab + 4a + 8 = 2b$$

$$\Rightarrow 4a + 8 = 2b$$

(divide throughout by 2)

$$\Rightarrow 2a + 4 = b$$

$$\Rightarrow b = 2a + 4$$

$$\text{For } a = 1, b = 2 \times 1 + 4 = 6$$

$$ab = 1 \times 6 = 6$$

$$\text{and } (a + 2)(b - 4) = 3 \times 2 = 6$$

$$\text{Hence, } ab = (a + 2)(b - 4)$$

$$\text{Let } a = 2, \text{ then } b = 2 \times 2 + 4 = 8$$

$$\text{Let } a = 3, \text{ then } b = 2 \times 3 + 4 = 10$$

Three such pairs are 1 and 6, 2 and 8 and 3 and 10

4. Expand (i) $(a + ab - 3b^2)(4 + b)$, and (ii) $(4y + 7)(y + 11z - 3)$.

Sol. (i) Here, $(a + ab - 3b^2)(4 + b)$

$$= (4 + b)(a + ab - 3b^2)$$

$$= 4(a + ab - 3b^2) + b(a + ab - 3b^2)$$

$$= 4a + 4ab - 12b^2 + ab + ab^2 - 3b^3$$

$$= 4a + 5ab - 12b^2 + ab^2 - 3b^3$$

(ii) Here, $(4y + 7)(y + 11z - 3)$

$$= 4y(y + 11z - 3) + 7(y + 11z - 3)$$

$$= 4y^2 + 44yz - 12y + 7y + 77z - 21$$

$$= 4y^2 + 44yz - 5y + 77z - 21$$

5. Expand (i) $(a - b)(a + b)$, (ii) $(a - b)(a^2 + ab + b^2)$ and (iii) $(a - b)(a^3 + a^2b + ab^2 + b^3)$, Do you see a pattern? What would be the next identity in the pattern that you see? Can you check it by expanding?

Sol. (i) Here, $(a - b)(a + b)$

$$= a(a + b) - b(a + b)$$

$$= a^2 + ab - ab - b^2$$

$$= a^2 - b^2$$

(ii) Here, $(a - b)(a^2 + ab + b^2)$

$$= a(a^2 + ab + b^2) - b(a^2 + ab + b^2)$$

$$= a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3$$

$$= a^3 - b^3$$

(iii) we have, $(a - b)(a^3 + a^2b + ab^2 + b^3)$

$$= a(a^3 + a^2b + ab^2 + b^3) - b(a^3 + a^2b + ab^2 + b^3)$$

$$= a^4 + a^3b + a^2b^2 + ab^3 - a^3b - a^2b^2 - ab^3 - b^4$$

$$= a^4 - b^4$$

We observe the following pattern $(a - b)(a^n + a^{n-1}b + \dots + b^n) = a^{n+1} - b^{n+1}$

Next identity in the pattern would be $(a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4) = a^5 - b^5$

FIGURE IT OUT-02

1. Which is greater: $(a - b)^2$ or $(b - a)^2$? Justify your answer.

Sol. Here, $(a - b)^2 = a^2 + b^2 - 2ab$... (1)

and $(b - a)^2 = b^2 + a^2 - 2ab$

$b^2 + a^2 = a^2 + b^2$ and $ba = ab$

$(b - a)^2 = a^2 + b^2 - 2ab$... (2)

Comparing (1) and (2), we get $(a - b)^2 = (b - a)^2$

2. Express 100 as the difference of two squares.

Sol. $a^2 - b^2 = 100$

$$(a + b)(a - b) = 100$$

$$[100 = 1 \times 100, 2 \times 50, 4 \times 25, 5 \times 20, 10 \times 10]$$

We can take anyone

$$\text{Let us take } 50 \times 2 = 100$$

Hence,

$$(a + b)(a - b) = 50 \times 2$$

$$a + b = 50 \quad \dots (1)$$

$$a - b = 2 \quad \dots (2)$$

Adding (1) and (2)

$$2a = 52$$

$$\Rightarrow a = 26$$

Substituting $a = 26$ in (1)

$$26 + b = 50$$

$$\Rightarrow b = 50 - 26 = 24$$

$$\text{Let us check } 26^2 - 24^2 = 676 - 576 = 100$$

$$\text{Hence, } 26^2 - 24^2 = 100$$

3. Find 406^2 , 72^2 , 145^2 , 1097^2 , and 124^2 using the identities you have learnt so far.

Sol. (i) $406^2 = (400 + 6)^2$

$$= 400^2 + 2 \times 400 \times 6 + 6^2$$

$$= 160000 + 4800 + 36$$

$$= 164836$$

(ii) $72^2 = (50 + 22)^2$

$$= 50^2 + 2 \times 50 \times 22 + 22^2$$

$$= 2500 + 2200 + 484$$

$$= 5184$$

(iii) $145^2 = (150 - 5)^2$

$$= 150^2 - 2 \times 150 \times 5 + 5^2$$

$$= 22500 - 1500 + 25$$

$$= 21025$$

(iv) $1097^2 = (1100 - 3)^2$

$$= 1100^2 - 2 \times 1100 \times 3 + 3^2$$

$$= 1210000 - 6600 + 9$$

$$= 1203409$$

(v) $124^2 = (100 + 24)^2$

$$= 100^2 + 2 \times 100 \times 24 + 24^2$$

$$= 10000 + 4800 + 576$$

$$= 15376$$

4. Do Patterns 1 and 2 hold only for counting numbers? Do they hold for negative integers as well? What about fractions? Justify your answer.

Sol. $2(a^2 + b^2) = (a + b)^2 + (a - b)^2$

Case-I:

Let

$$a = 4, b = 2$$

$$\text{LHS} = 2(4^2 + 2^2) = 2 \times (16 + 4) = 40$$

$$\text{RHS} = (4 + 2)^2 + (4 - 2)^2 = 6^2 + 2^2 = 36 + 4 = 40$$

$\therefore$ Pattern 1 holds for counting numbers.

Case-II

Let

$$a = -4, b = -2$$

$$\text{LHS} = 2((-4)^2 + (-2)^2)$$

$$= 2 \times (16 + 4)$$

$$= 2 \times 20$$

$$= 40$$

$$\text{RHS} = (-4 + (-2))^2 + (-4 - (-2))^2$$

$$= (-4 - 2)^2 + (-4 + 2)^2$$

$$= (-6)^2 + (-2)^2$$

$$= 36 + 4$$

$$= 40$$

$$\text{LHS} = \text{RHS}$$

$\therefore$ Pattern 1 holds for negative integers also.

Case III

$$\text{Let } a = \frac{1}{2}, b = \frac{1}{3}$$

$$\text{LHS} = 2 \left(\left(\frac{1}{2} \right)^2 + \left(\frac{1}{3} \right)^2 \right)$$

$$= 2 \left(\frac{1}{4} + \frac{1}{9} \right) = 2 \times \frac{13}{36} = \frac{13}{18}$$

$$\text{RHS} = \left(\frac{1}{2} + \frac{1}{3} \right)^2 + \left(\frac{1}{2} - \frac{1}{3} \right)^2$$

$$= \left(\frac{5}{6} \right)^2 + \left(\frac{1}{6} \right)^2 = \frac{25}{36} + \frac{1}{36}$$

$$= \frac{26}{36} = \frac{13}{18}$$

The pattern holds for fractions also.

Pattern 2

$$a^2 - b^2 = (a + b)(a - b)$$

Case I

$$\text{Let } a = 5, b = 3$$

$$\text{LHS} = 5^2 - 3^2 = 25 - 9 = 16$$

$$\text{RHS} = (5 + 3)(5 - 3) = 8 \times 2 = 16$$

$$\therefore \text{LHS} = \text{RHS}$$

$\therefore$ Pattern 2 holds for counting numbers

Case II

$$\text{Let } a = -5, b = -3$$

$$\text{Now, LHS} = (-5)^2 - (-3)^2 = 25 - 9 = 16$$

$$\text{And RHS} = [(-5) + (-3)][(-5) - (-3)]$$

$$= (-5 - 3)(-5 + 3)$$

$$= (-8)(-2)$$

$$= 16$$

$$\therefore \text{LHS} = \text{RHS}$$

$\therefore$ Pattern 2 holds for negative integers also.

Case III

$$\text{Let } a = \frac{1}{2}, b = \frac{1}{3}$$

$$\text{LHS} = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{3}\right)^2$$

$$= \frac{1}{4} - \frac{1}{9}$$

$$= \frac{9-4}{36}$$

$$= \frac{5}{36}$$

$$\text{And RHS} = \left(\frac{1}{2} + \frac{1}{3}\right)\left(\frac{1}{2} - \frac{1}{3}\right) = \left(\frac{3+2}{6}\right)\left(\frac{3-2}{6}\right) = \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$$

$$\therefore \text{LHS} = \text{RHS}$$

$\therefore$ Pattern 2 holds for fractions also.

FIGURE IT OUT-03

1. Compute these products using the suggested identity.

(i) 46^2 using Identity 1A for $(a + b)^2$

(ii) 397×403 using Identity 1C for $(a + b)(a - b)$

(iii) 91^2 using Identity 1B for $(a - b)^2$

(iv) 43×45 using Identity 1C for $(a + b)(a - b)$

Sol. (i) $46^2 = (40 + 6)^2$

$$= 40^2 + 2 \times 40 \times 6 + 6^2 \quad [\because (a + b)^2 = a^2 + 2ab + b^2]$$

$$= 1600 + 480 + 36$$

$$= 2116$$

(ii) $397 \times 409 = (403 - 3)(400 + 3) \quad [\because (a + b)(a - b) = a^2 - b^2]$

$$= 400^2 - 3^2$$

$$= 160000 - 9$$

$$= 159991$$

(iii) $91^2 = (100 - 9)^2$

$$= 100^2 - 2 \times 100 \times 9 + 9^2 \quad [\because (a - b)^2 = a^2 + b^2 - 2ab]$$

$$= 10000 - 1800 + 81$$

$$= 8281$$

(iv) $43 \times 45 = (44 - 1)(44 + 1) \quad [\because a^2 - b^2 = (a + b)(a - b)]$

$$= 44^2 - 1^2$$

$$= 1936 - 1$$

$$= 1935$$

2. Use either a suitable identity or the distributive property to find each of the following products.

(i) $(p - 1)(p + 11)$

(ii) $(3a - 9b)(3a + 9b)$

(iii) $-(2y + 5)(3y + 4)$

(iv) $(6x + 5y)^2$

(v) $(2x - \frac{1}{2})^2$

(vi) $(7p) \times (3r) \times (p + 2)$

Sol. (i) $(p - 1)(p + 11)$
 $= p(p + 11) - 1(p + 11)$
 $= p^2 + 11p - p - 11$
 $= p^2 + 10p - 11$

(ii) $(3a - 9b)(3a + 9b)$
 $= (3a)^2 - (9b)^2$
 $= 9a^2 - 81b^2$

(iii) $-(2y + 5)(3y + 4)$
 $= (-2y - 5)(3y + 4)$
 $= -2y(3y + 4) - 5(3y + 4)$
 $= -6y^2 - 8y - 15y - 20$
 $= -6y^2 - 23y - 20$

(iv) $(6x + 5y)^2$
 $= (6x)^2 + 2(6x)(5y) + (5y)^2$
 $= 36x^2 + 60xy + 25y^2$

(v) $\left(2x - \frac{1}{2}\right)^2$
 $= (2x)^2 - 2 \times 2x \times \frac{1}{2} + \left(\frac{1}{2}\right)^2$
 $= 4x^2 - 2x + \frac{1}{4}$

(vi) $(7p) \times (3r) \times (p + 2)$
 $= 7p \times 3r \times (p + 2)$
 $= 21pr(p + 2)$
 $= 21pr \times p + 21pr \times 2$
 $= 21p^2r + 42pr$

3. For each statement identify the appropriate algebraic expression(s).

(i) Two more than a square number.

- $2 + s$
- $(s + 2)^2$
- $s^2 + 2$
- $s^2 + 4$
- $2s^2$
- 2^2s

(ii) The sum of the squares of two consecutive numbers

- $m^2 + n^2$
- $(m + n)^2$
- $m^2 + 1$
- $m^2 + (m + 1)^2$
- $m^2 + (m - 1)^2$
- $(m + (m + 1))^2$
- $(2m)^2 + (2m + 1)^2$

Sol. (i) Two more than a square is $s^2 + 2$

(ii) Sum of the squares of two consecutive numbers is $m^2 + (m + 1)^2$

4. Consider any 2 by 2 square of numbers in a calendar, as shown in the figure.

February						
Sun	Mon	Tue	Wed	Thr	Fri	Sat
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	

Find products of numbers lying along each diagonal — $4 \times 12 = 48$, $5 \times 11 = 55$. Do this for the other 2 by 2 squares. What do you observe about the diagonal products? Explain why this happens.

Hint: Label the numbers in each 2 by 2 square as

a	$(a + 1)$
$a + 7$	$(a + 8)$

Sol. Case -I

6	7
13	14

Here, $6 \times 14 = 84$

$13 \times 7 = 91$

Difference = $91 - 84 = 7$

Case II

9	10
16	17

Here, $9 \times 17 = 153$

$16 \times 10 = 160$

Difference = $160 - 153 = 7$

We observe that the difference of the diagonal products in both cases is always 7.

5. Verify which of the following statements are true.

(i) $(k + 1)(k + 2) - (k + 3)$ is always a multiple of 2.

(ii) $(2q + 1)(2q - 3)$ is a multiple of 4.

(iii) Squares of even numbers are multiples of 4, and squares of odd numbers are 1 more than multiples of 8.

(iv) $(6n + 2)^2 - (4n + 3)^2$ is 5 less than a square number.

Sol.: (i) $(k + 1)(k + 2) - (k + 3)$ is always a multiple of 2

Let $k = 5$, Then $(5 + 1)(5 + 2) - (5 + 3)$

$$= 6 \times 7 - 8$$

$$= 42 - 8$$

$$= 34$$

34 is a multiple of 2.

$\therefore$ The statement is true.

(ii) $(2q + 1)(2q - 3)$ is a multiple of 4.

Let, $q = 3$, Then $(6 + 1)(6 - 3)$

$$= 7 \times 3$$

$$= 21$$

21 is not a multiple of 4

$\therefore$ The statement is false.

(iii) The square of an even number is a multiple of 4.

$$2^2 = 4 = 4 \times 1$$

$$4^2 = 16 = 4 \times 4$$

$$6^2 = 36 = 4 \times 9$$

$\therefore$ The statement is true.

The square of an odd number is 1 more than a multiple of 8.

$$3^2 = 9 = 8 \times 1 + 1$$

$$5^2 = 25 = 8 \times 3 + 1$$

$$7^2 = 49 = 8 \times 6 + 1$$

$\therefore$ The statement is true.

(iv) $(6n + 2)^2 - (4n + 3)^2$ is 5 less than a square number.

$$\text{Let } n = 2, (6 \times 2 + 2)^2 - (4 \times 2 + 3)^2$$

$$= 14^2 - 11^2$$

$$= 196 - 121$$

$$= 75$$

$$= 80 - 5$$

But 80 is not a square number.

$\therefore$ The statement is false.

6. A number leaves a remainder of 3 when divided by 7, and another number leaves a remainder of 5 when divided by 7. What is the remainder when their sum, difference, and product are divided by 7?

Sol. Let the numbers be x and y .

$$x = 7a + 3,$$

$$y = 7b + 5$$

$$\text{Sum} = x + y$$

$$= 7a + 3 + 7b + 5$$

$$= 7(a + b) + 8$$

$$= 7(a + b) + 7 + 1$$

$$= 7(a + b + 1) + 1$$

$\therefore$ The remainder on division by 7 is 1.

$$\text{Difference} = x - y$$

$$= (7a + 3) - (7b + 5)$$

$$= 7a + 3 - 7b - 5$$

$$= 7(a - b) - 2$$

$$= 7(a - b) - 1 + 5 \quad (\because -2 = -7 + 5)$$

$$= 7(a - b - 1) + 5$$

$\therefore$ The remainder on division by 7 is 5.

$$\text{Product} = xy$$

$$= (7a + 3)(7b + 5)$$

$$= 49ab + 35a + 21b + 15$$

$$= (49ab + 35a + 21b + 14) + 1$$

$$= 7(7ab + 5a + 3b + 2) + 1$$

$\therefore$ The remainder on division by 7 is 1.

7. Choose three consecutive numbers, square the middle one, and subtract the product of the other two. Repeat the same with other sets of numbers. What pattern do you notice? How do we write this as an algebraic equation? Expand both sides of the equation to check that it is a true identity.

Sol. Let us take the numbers 7, 8, 9

Now,

$$8^2 - 7 \times 9 = 64 - 63 = 1$$

Let us take the numbers 10, 11, 12

Then

$$11^2 - 10 \times 12 = 121 - 120 = 1$$

Generalizing:

Let the numbers be $a - 1$, a , $a + 1$

Then

$$a^2 - (a + 1)(a - 1) = 1$$

LHS

$$= a^2 - (a + 1)(a - 1)$$

$$= a^2 - (a^2 - 1)$$

$$= a^2 - a^2 + 1$$

$$= 1$$

$$\therefore \{LHS = RHS\}$$

Hence, the identity is correct.

8. What is the algebraic expression describing the following steps: add any two numbers? Multiply this by half of the sum of the two numbers? Prove that this result will be half of the square of the sum of the two numbers.

Sol. Let the two numbers be a and b .

Step 1: $a + b$

Step 2: $(a + b) \times \frac{1}{2} (a + b)$

$$\therefore (a + b) \times \frac{1}{2} (a + b) = \frac{1}{2} (a + b)^2$$

9. Which is larger? Find out without fully computing the product.

(i) 14×26 or 16×24

(ii) 25×75 or 26×74

Sol. (i) Let $p = 14 \times 26$

$$p' = 16 \times 24$$

$$= (14 + 2)(26 - 2)$$

$$= 14 \times 26 + 2 \times 26 - 14 \times 2 - 2 \times 2$$

$$= 14 \times 26 + 2(26 - 14 - 2)$$

$$= 14 \times 26 + 2 \times 10$$

$$p' = p + 2 \times 10$$

$$\therefore p' > p \text{ or } 16 \times 24 > 14 \times 26$$

(ii) Let

$$p = 25 \times 75$$

$$p' = 26 \times 74$$

$$= (25 + 1)(75 - 1)$$

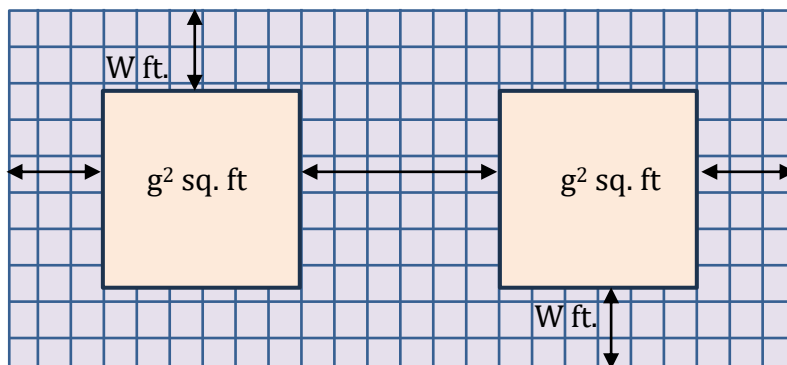
$$= 25 \times 75 + 75 \times 1 - 25 \times 1 - 1 \times 1$$

$$= p + (75 - 25 - 1)$$

$$= p + 49$$

$$\therefore p' > p \text{ or } 26 \times 74 > 25 \times 75$$

10. A tiny park is coming up in Dhauli. The plan is shown in the figure. The two square plots, each of area g^2 sq. ft., will have a green cover. All the remaining area is a walking path w ft. wide that needs to be tiled. Write an expression for the area that needs to be tiled.



Sol. Length = $w + g + 2w + g + w = 4w + 2g$

$$\text{Breadth} = w + g + w = 2w + g$$

$$\text{Area of park} = (4w + 2g)(2w + g)$$

$$= 8w^2 + 4wg + 4wg + 2g^2$$

$$= 8w^2 + 8wg + 2g^2$$

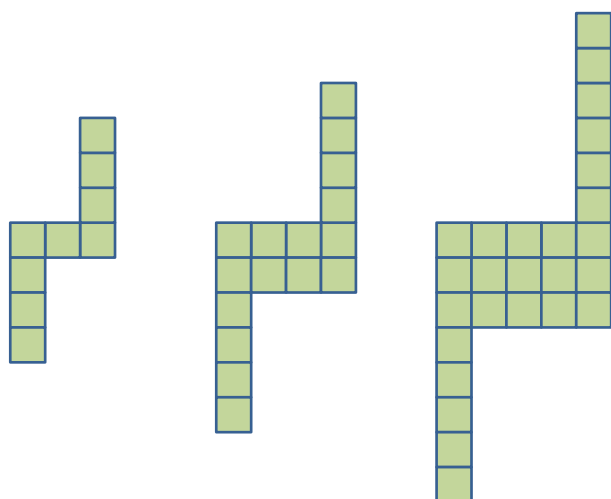
$$\text{Area of path} = \text{Area of park} - \text{Area of green cover}$$

$$= 8w^2 + 8wg + 2g^2 - 2g^2$$

$$= 8w^2 + 8wg$$

$$\therefore (8w^2 + 8wg) \text{ sq. feet area needs to be tiled.}$$

11. For each pattern shown below,
- Draw the next figure in the sequence.
 - How many basic units are there in Step 10?
 - Write an expression to describe the number of basic units in Step y .



Step 1

Step 2

Step 3



Step 1

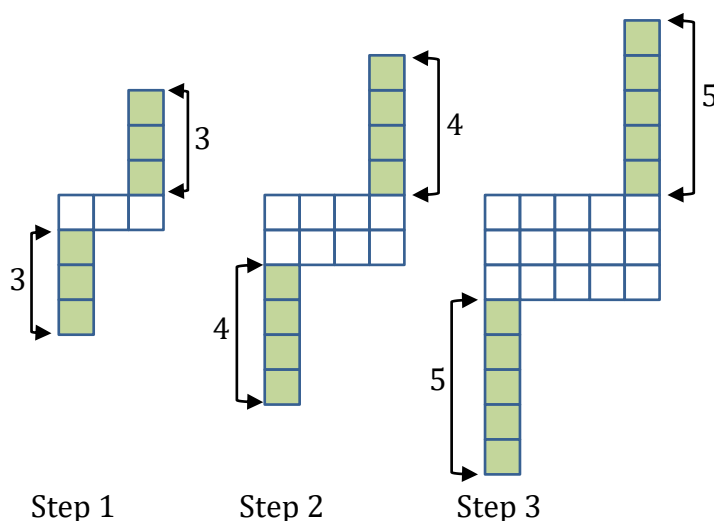


Step 2



Step 3

Sol.: (a)



Step 1

Step 2

Step 3

Step 1: 2 vertical strips of 3 units each + 1 vertical strip of 3 units

= 3 strips of 3 units each

= 9 units squares

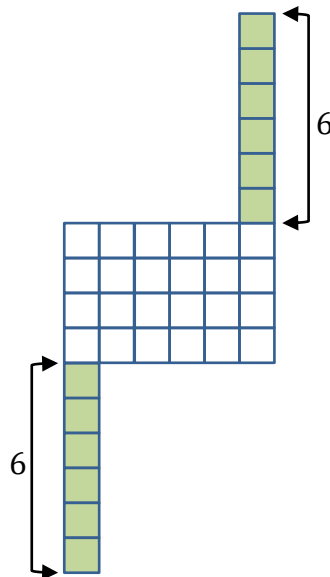
= $(1 + 2)^2$ units squares

Step 2: 4 strips of 4 units each

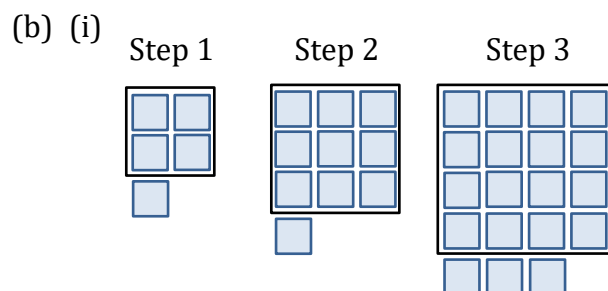
= 16 units squares

= $(2 + 2)^2$ units squares

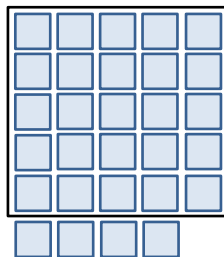
Step 3: 5 strips of 5 units each
 = 25 units squares
 = $(3 + 2)^2$ units squares



Step 4: (i) 6 strips of 6 units each = 2 are vertical and 4 are horizontal.
 (ii) Number of unit squares in step 10 = $(10 + 2)^2 = 144$
 (iii) Number of unit squares in step $y = (y + 2)^2$



Number of unit squares in step 1 = $5 = 2^2 + 1$
 Number of unit squares in step 2 = $11 = 3^2 + 2$
 Number of unit squares in step 3 = $19 = 4^2 + 3$



(ii) Step 1 has $(1 + 1)^2 + 1$ or 5 squares
 Step 2 has $(2 + 1)^2 + 2$ or 11 squares
 Step 3 has $(3 + 1)^2 + 3$ or 19 squares
 Hence step 10 has $(10 + 1)^2 + 10$ or 131 squares
 (iii) Step y has $[(y + 1)^2 + y]$ squares