

NCERT QUESTIONS WITH SOLUTIONS

FIGURE IT OUT-01

1. The sum of four consecutive numbers is 34. What are these numbers?

Sol. Let x , $x + 1$, $x + 2$, and $x + 3$ be the four consecutive numbers, respectively.

$$\therefore x + (x + 1) + (x + 2) + (x + 3) = 34$$

$$\Rightarrow 4x + 6 = 34$$

$$\Rightarrow 4x = 34 - 6$$

$$\Rightarrow 4x = 28$$

$$\Rightarrow x = 7$$

$$\therefore x + 1 = 7 + 1 = 8$$

$$x + 2 = 7 + 2 = 9$$

$$\text{and } x + 3 = 7 + 3 = 10$$

Thus, the four consecutive numbers are 7, 8, 9, and 10.

2. Suppose p is the greatest of five consecutive numbers. Describe the other four numbers in terms of p .

Sol. Given p is the greatest of five consecutive numbers.

The other four numbers in terms of p are $(p - 1)$, $(p - 2)$, $(p - 3)$, and $(p - 4)$.

$p - 1$ is the second largest number

$p - 2$ is the third largest number

$p - 3$ is the second smallest number

$p - 4$ is the smallest number

$$\therefore p > (p - 1) > (p - 2) > (p - 3) > (p - 4).$$

3. For each statement below, determine whether it is always true, sometimes true, or never true. Explain your answer. Mention examples and non-examples as appropriate. Justify your claim using algebra.

(i) The sum of two even numbers is a multiple of 3.

(ii) If a number is not divisible by 18, then it is also not divisible by 9.

(iii) If two numbers are not divisible by 6, then their sum is not divisible by 6.

(iv) The sum of a multiple of 6 and a multiple of 9 is a multiple of 3.

(v) The sum of a multiple of 6 and a multiple of 3 is a multiple of 9.

Sol. (i) Sometimes true, the sum of two even numbers is a multiple of 3.

Examples:

$2 + 4 = 6$, $8 + 10 = 18$, $14 + 16 = 30$ are multiples of 3.

$2 + 6 = 8$, $4 + 10 = 14$ are not multiples of 3. (ii) Sometimes true, if a number is not divisible by 18, then it is also not divisible by 9.

Examples:

27 is not divisible by 18, but 27 is divisible by 9. True

40 is not divisible by 18, also it is not divisible by 9. False

(iii) Never true, if two numbers are not divisible by 6, then their sum is not divisible by 6.

Examples:

8 and 10 are not divisible by 6.

The sum of two numbers $= 8 + 10 = 18$, is divisible by 6.

10 and 13 are not divisible by 6.

The sum of 10 and 13 $= 10 + 13 = 23$, which is not divisible by 6.

(iv) Always true

Multiple of 6; $6m$

Multiple of 9; $9n$

$$6m + 9n = 3(2m + 3n)$$

Hence multiple of 3.

- (v) Sometimes true, the sum of a multiple of 6 and a multiple of 3 is a multiple of 9.

Multiples of 6 are: 6, 12, 18, 24, 30,...

$6 + 12 = 18$ is a multiple of 9.

$12 + 18 = 30$ is not a multiple of 9.

$18 + 24 = 42$ is not a multiple of 9.

Sometimes true.

Multiples of 3 are: 3, 6, 9, 12, 15, 18,...

$3 + 6 = 9$ is a multiple of 9.

$6 + 9 = 15$ is not a multiple of 9.

Sometimes true.

4. Find a few numbers that leave a remainder of 2 when divided by 3 and a remainder of 2 when divided by 4. Write an algebraic expression to describe all such numbers.

Sol. Here, Remainder = 2, Dividend = 3

$\therefore$ Number = (Quotient $\times$ Dividend) + Remainder = $(K \times 3) + 2$

where, $K = 1, 2, 3, \dots$

Numbers = $1 \times 3 + 2 = 3 + 2 = 5$

Numbers = $2 \times 3 + 2 = 6 + 2 = 8$

Numbers = $3 \times 3 + 2 = 9 + 2 = 11$

Thus, 5, 8, and 11 are numbers that leave a remainder of 2 when divided by 3.

Algebraic expression = $3K + 2$

Here, Remainder = 2, dividend = 4

Number = $4K + 2$, where $K = 1, 2, 3, 4, \dots$

Numbers = $4 \times 1 + 2 = 4 + 2 = 6$

Numbers = $4 \times 2 + 2 = 8 + 2 = 10$

Numbers = $4 \times 3 + 2 = 12 + 2 = 14$

Algebraic expression = $4K + 2$

Thus, 6, 10, and 14 are numbers that leave a remainder of 2 when divided by 4.

5. "I hold some pebbles, not too many, when I group them in 3's, one stays with me. Try pairing them up — it simply won't do. A stubborn odd pebble remains in my view. Group them by 5, yet one's still around, but grouping by seven, perfection is found. More than one hundred would be far too bold. Can you tell me the number of pebbles I hold?"



Sol. The LCM of 3, 5, and 7 = $3 \times 5 \times 7 = 105$
[$\because$ 3, 5, and 7 are prime numbers]

No. of pebbles = $105 + 1 = 106$,

6. Tathagat has written several numbers that leave a remainder of 2 when divided by 6. He claims, "If you add any three such numbers, the sum will always be a multiple of 6." Is Tathagat's claim true?

Sol. The expression has been written by Tathagat = $6k + 2$

where, $k = 1, 2, 3, 4, 5, 6, \dots$

$6 \times 1 + 2 = 8$

$6 \times 2 + 2 = 14$

$6 \times 3 + 2 = 20$

$6 \times 4 + 2 = 26$

The sum of three numbers

$8 + 14 + 20 = 42$, it is a multiple of 6.

$14 + 20 + 26 = 60$, it is a multiple of 6.

Yes, Tathagat's claim is true.

7. When divided by 7, the number 661 leaves a remainder of 3, and 4779 leaves a remainder of 5. Without calculating, can you say what remainders the following expressions will leave when divided by 77? Show the solution both algebraically and visually.

(i) $4779 + 661$

(ii) $4779 - 661$

Sol. Given, $661 = K \times 7 + 3$,
where $K = 1, 2, 3, 4, \dots$
and, $4779 = K \times 7 + 5$

Algebraic Method:

(i) $4779 + 661$

$$4779 = (682 \times 7) + 5$$

$$\text{Remainder} = 5$$

$$661 = (94 \times 7) + 3$$

$$\text{Remainder} = 3$$

$$\therefore 4779 + 661 = 5 + 37 = 87$$

$$= 1 = \text{Remainder}$$

(ii) $4779 - 661$

$$\therefore 4779 - 661 = 5 - 17 = 87$$

$$= 4 = \text{Remainder}$$

Visualization Method:

$$\begin{aligned} \text{(i)} \quad 4779 + 661 &= (682 \times 7) + 5 + (94 \times 7) + 3 \\ &= 7 \times (682 + 94) + 5 + 3 \\ &= 7 \times 776 + 8 \\ &= \text{Divisible by 7} + 87 \\ &= 1, \text{Remainder} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad 4779 - 661 &= (682 \times 7) + 5 - (94 \times 7) - 3 \\ &= 7 \times (682 - 94) + 5 - 3 \\ &= 7 \times 588 + 2 \\ &= \text{Divisible by 7} + 2 \\ &= 2, \text{Remainder} \end{aligned}$$

8. Find a number that leaves a remainder of 2 when divided by 3, a remainder of 3 when divided by 4, and a remainder of 4 when divided by 5. What is the smallest such number? Can you give a simple explanation of why it is the smallest?

Sol. The expression of a number that leaves a remainder of 2 when divided by 3.

$$\text{Number} = 3K + 2$$

$$3 \times 1 + 2 = 5$$

$$3 \times 2 + 2 = 8$$

$$3 \times 3 + 2 = 11$$

$$3 \times 4 + 2 = 14$$

$$3 \times 5 + 2 = 17$$

$$3 \times 6 + 2 = 20$$

The expression of a number that leaves a remainder of 3 when divided by 4.

$$\text{Number} = 4K + 3$$

$$4 \times 1 + 3 = 7$$

$$4 \times 2 + 3 = 11$$

$$4 \times 3 + 3 = 15$$

$$4 \times 4 + 3 = 19$$

The expression of a number that leaves a remainder of 4 when divided by 5.

$$\text{Number} = 5K + 4$$

$$5 \times 1 + 4 = 9$$

$$5 \times 2 + 4 = 14$$

$$5 \times 3 + 4 = 19$$

$$5 \times 4 + 4 = 24$$

$$\begin{aligned} \text{Smallest number} &= \text{LCM of } (3, 4, 5) - 1 \\ &= 60 - 1 \\ &= 59 \end{aligned}$$

59 is the smallest number that leaves a remainder of 2 when divided by 3, a remainder of 3 when divided by 4, and a remainder of 4 when divided by 5.

FIGURE IT OUT-02

1. Find, without dividing, whether the following numbers are divisible by 9.

- (i) 123 (ii) 405
(iii) 8888 (iv) 93547
(v) 358095

Sol. If the sum of the digits of a number is divisible by 9, then the number is divisible by 9.

(i) Sum of the digits = $1 + 2 + 3 = 6$, is not divisible by 9.

Thus, 123 is not divisible by 9.

(ii) Sum of the digits = $4 + 0 + 5 = 9$, is divisible by 9.

Thus, 405 is divisible by 9.

(iii) Sum of the digits = $8 + 8 + 8 + 8 = 32$, is not divisible by 9.

Thus, 8888 is not divisible by 9.

(iv) Sum of the digits = $9 + 3 + 5 + 4 + 7 = 28$, is not divisible by 9. Thus 93547 is not divisible by 9.

(v) Sum of the digits = $3 + 5 + 8 + 0 + 9 + 5 = 30$, is not divisible by 9.

Hence, 358095 is not divisible by 9.

2. Find the smallest multiple of 9 with no odd digits.

Sol. Multiples of 9 = 9, 18, 27, 36,..., 288,.....
The smallest multiple of 9 with an odd digit is 9.

The smallest multiple of 9 that can be formed by summing even digits is 18 (since 9 is odd).

Thus, the smallest multiple of 9 with no odd digits is 288.

3. Find the multiple of 9 that is closest to the number 6000.

Sol. Given, 6000

Sum of the digits = $6 + 0 + 0 + 0 = 6$

We know that, if the number is divisible by 9, then the sum of the digits is divisible by 9.

If we add 3 to the number 6000.

$6000 + 3 = 6003$, it is divisible by 3.

Thus, the multiple of 9 that is closest to the number is 6003.

4. How many multiples of 9 are there between the numbers 4300 and 4400?

Sol. The multiples of 9 are there between the numbers 4300 and 4400 are 4302, 4311, 4320,....., 4392

The number of multiples of 9 =

Last term – First term

Difference

$$= \frac{4392 - 4302}{9} + 1$$

$$= \frac{90}{9} + 1$$

$$= 10 + 1$$

$$= 11$$

Thus, the multiples of 9 are 11.

FIGURE IT OUT-03

1. The digital root of an 8-digit number is 5. What will be the digital root of 10 more than that number?

Sol. Consider the 8-digit number 80000006.

The digital root of 80000006 = $8 + 0 + 0 + 0 + 0 + 0 + 0 + 6$

$$= 14$$

$$= 1 + 4$$

$$= 5$$

$$10 \text{ more than } 80000006 = 80000006 + 10 = 80000016$$

The digital root of 80000016 = $8 + 0 + 0 + 0 + 0 + 0 + 1 + 6$

$$= 15$$

$$= 1 + 5$$

$$= 6$$

Thus, the digital root of 10 more than 80000006 is 6.

2. Write any number. Generate a sequence of numbers by repeatedly adding 11. What would be the digital roots of this sequence of numbers? Share your observations.

Sol. Consider the number = 40

The sequence of numbers by repeatedly adding 11 are 40, 51(40 + 11), 62(51 + 11), 73(62 + 11), 84(73 + 11), 95(84 + 11), 106(95 + 11), 117(106 + 11), 128(117 + 11), 139(128 + 11), etc.

The digital roots of this sequence of numbers are:

$$40 = 4 + 0 = 4;$$

$$51 = 5 + 1 = 6;$$

$$62 = 6 + 2 = 8;$$

$$73 = 7 + 3 = 10 = 1 + 0 = 1;$$

$$84 = 8 + 4 = 12 = 1 + 2 = 3;$$

$$95 = 9 + 5 = 14 = 1 + 4 = 5;$$

$$106 = 1 + 0 + 6 = 7;$$

$$117 = 1 + 1 + 7 = 9;$$

$$128 = 1 + 2 + 8 = 11 = 1 + 1 = 2;$$

$$139 = 1 + 3 + 9 = 13 = 1 + 3 = 4, \dots \text{ etc.}$$

Thus, the digital roots of this sequence of numbers are 4, 6, 8, 1, 3, 5, 7, 9, 2, 4,

Observations:

The digital roots are 4, 6, 8, 1, 3, 5, 7, 9, 2, 4,

This sequence starts repeating after 9 steps.

So the digital roots form a cycle: 4, 6, 8, 1, 3, 5, 7, 9, 2, 4,

3. What will be the digital root of the number $9a + 36b + 13$?

Sol. First Method:

The digital root of the number $9a + 36b + 13 = 9a + 36b + 9 + 4$

$$= 9(a + 4b + 1) + 4$$

$$= 9 + 4$$

$= 13$ [$\because$ The digital root of multiples of 9 is always 9.]

$$= 1 + 3 = 4$$

Thus, the digital root of the number $9a + 36b + 13$ will be 4.

Second Method:

We have $9a + 36b + 13$

Here, a and b are integers

Put $a = 1, b = 1,$

$$9a + 36b + 13 = 9 \times 1 + 36 \times 1 + 13$$

$$= 9 + 36 + 13$$

$$= 58$$

The digital root of $58 = 5 + 8 = 13 = 1 + 3$

$$= 4$$

Put $a = 2, b = 3,$

$$9a + 36b + 13 = 9 \times 2 + 36 \times 3 + 13$$

$$= 18 + 108 + 13$$

$$= 139$$

The digital root of $139 = 1 + 3 + 9 = 13 = 1 + 3$

$$+ 3 = 4$$

Thus, the expression $9a + 36b + 13$ always has a digital root of 4.

4. Make conjectures by examining if there are any patterns or relations between

(i) the parity of a number and its digital root.

(ii) the digital root of a number and the remainder obtained when the number is divided by 3 or 9.

Sol. Consider the pattern: 8, 16, 24, 32, 40,

(i) $8 = 8 = \text{digital root, parity} \rightarrow \text{even}$

$$16 = 1 + 6 = 7 = \text{digital root,}$$

parity $\rightarrow \text{odd}$

$$24 = 2 + 4 = 6 = \text{digital root,}$$

parity $\rightarrow \text{even}$

$$32 = 3 + 2 = 5 = \text{digital root,}$$

parity $\rightarrow \text{odd}$

$$40 = 4 + 0 = 4 = \text{digital root,}$$

parity $\rightarrow \text{even}$

(ii) Divided by 3

$$8 \div 3 \Rightarrow 2, \text{ Remainder}$$

$$24 \div 3 \Rightarrow 0, \text{ Remainder}$$

$$32 \div 3 \Rightarrow 2, \text{ Remainder}$$

$$40 \div 3 \Rightarrow 1, \text{ Remainder}$$

Divided by 9

$$8 \div 9 \Rightarrow 8, \text{ Remainder}$$

$$24 \div 9 \Rightarrow 6, \text{ Remainder}$$

$$32 \div 9 \Rightarrow 5, \text{ Remainder}$$

$$40 \div 9 \Rightarrow 4, \text{ Remainder}$$

FIGURE IT OUT-04

1. If $31z5$ is a multiple of 9, where z is a digit, what is the value of z ? Explain why there are two answers to this problem.

Sol. Here, $31z5$

$$\text{Sum of the digits} = 3 + 1 + z + 5 = 9 + z$$

$(9 + z)$ should be divisible by 9.

$z = 0$, 3105 is divisible by 9.

$z = 9$, 3195 is also divisible by 9.

$\therefore z = 0$ or 9

There are two answers to this problem because, excluding z , the sum of the digits is divisible by 9.

2. "I take a number that leaves a remainder of 8 when divided by 12. I take another number, which is 4 short of a multiple of 12. Their sum will always be a multiple of 8", claims Snehal. Examine his claim and justify your conclusion.

Sol. A number that leaves a remainder of 8. when divided by 12: $12k + 8$, where $k \geq 1$. Also, another number 4 short of a multiple of 12: $12k - 4$

3. When is the sum of two multiples of 3, a multiple of 6, and when is it not? Explain the different possible cases, and generalise the pattern.

Sol. Multiples of 3 are: 3, 6, 9, 12, 15, 18,.....

$$3 + 6 = 9, \text{ not a multiple of 6.}$$

$$6 + 9 = 15, \text{ not a multiple of 6.}$$

$$3 + 9 = 12, \text{ multiple of 6.}$$

$$6 + 12 = 18, \text{ multiple of 6.}$$

There are two possible cases.

- If both numbers are odd, then the sum is a multiple of 6.
- If both numbers are even, then the sum is a multiple of 6.

4. Sreelatha says, "I have a number that is divisible by 9. If I reverse its digits, it will still be divisible by 9".

(i) Examine if her conjecture is true for any multiple of 9.

(ii) Are any other digit shuffles possible such that the number formed is still a multiple of 9?

Sol. We know that if the sum of the digits of a number is divisible by 9, then the numbers is also divisible by 9.

If we reverse its digits, the sum of its digits will not change because the digits are the same. So, the reverse is also divisible by 9.

(i) Let the number be 81.

So, $81 \rightarrow 8 + 1 = 9$, the number is divisible by 9.

Reverse of 81 is 18.

So, $18 \rightarrow 1 + 8 = 9$, the number is divisible by 9.

(ii) Yes, any other digit shuffles is possible as the number is still a multiple of 9.

Let take a number 459

So, $459 \rightarrow (4 + 5 + 9) \rightarrow 18$, the number is divisible by 9.

Reverse of 459 is 954.

So, $954 \rightarrow 9 + 5 + 4 = 18$, the number is still divisible by 9.

5. If $48a23b$ is a multiple of 18, list all possible pairs of values for a and b .

Sol. Given by question,
 $48a23b$ is a multiple of 18.
 As we know that,
 If the number is a multiple of 18, then it is also a multiple of 2 and 9.
 $\therefore 48a23b$
 Sum of the digits = $4 + 8 + a + 2 + 3 + b = 17 + a + b$
 Case 1: Put $a = 1$ and $b = 0$
 481230 , it is possible values of a and b .
 Sum = 18, it is divisible by 9.
 Case 2: Put $a = 4$ and $b = 6$
 484236
 Sum = $17 + 10 = 27$, it is divisible by 9.
 Thus, the possible values of a and b are $a = 1$ and $b = 0$, $a = 4$ and $b = 6$; there are two possible cases.

6. If $3p7q8$ is divisible by 44, list all possible pairs of values for p and q .

Sol. Given by question, $3p7q8$ is divisible by 44.
 As we know, if a number is divisible by 44, then it is also divisible by 4 and 11.
 $\therefore 3p7q8$
 Case 1: Put $p = 1$ and $q = 0$
 37708 is divisible by 4 and 11, then it is also divisible by 44.
 Case 2: Put $p = 5$ and $q = 2$
 35728 is divisible by 4 and 11, then it is also divisible by 44.
 Case 3: Put $p = 3$ and $q = 4$
 33748 is divisible by 4 and 11, then it is also divisible by 44.
 Case 4: Put $p = 1$ and $q = 6$
 31768 is divisible by 4 and 11, then it is also divisible by 11.
 Thus, $(p = 7, q = 0)$, $(p = 5, q = 2)$, $(p = 3, q = 4)$, and $(p = 1$ and $q = 6)$ are the possible pairs of values for p and q .

7. Find three consecutive numbers such that the first number is a multiple of 2, the second number is a multiple of 3, and the third number is a multiple of 4. Are there more such numbers? How often do they occur?

Sol. Let x , $x + 1$ and $(x + 2)$ be the three numbers
 Put $x = 2, \Rightarrow 2, 3, 4$
 Put $x = 14, \Rightarrow 14, 15, 6$
 Put $x = 26, \Rightarrow 26, 27, 28$
 Put $x = 38, \Rightarrow 38, 39, 40$
 Thus, the three consecutive numbers are $(14, 15, 16)$,
 Put $x = 26, \Rightarrow 26, 27, 28$
 $(26, 27, 28)$ and $(38, 39, 40)$
 There are infinite numbers, spaced apart by 12.

8. Write five multiples of 36 between 45,000 and 47,000. Share your approach with the class.

Sol. We know that if a number is a multiple of 36, then it is also a multiple of 4 and 9.
 45000
 Last two digits = 00, it is divisible by 4.
 Sum of the digits = $4 + 5 + 0 + 0 + 0 = 9$, it is also divisible by 9.
 Thus, 45000 is completely divisible by 36.
 The five multiples of 36 between 45,000 and 47,000.
 $(45,000 + 36)$, $(45,000 + 2 \times 36)$, $(45,000 + 3 \times 36)$, $(45,000 + 4 \times 36)$ and $(45,000 + 5 \times 36)$
 i.e., 45,036, 45,072, 45,108, 45,144, and 45,180.

9. The middle number in the sequence of 5 consecutive even numbers is $5p$. Express the other four numbers in sequence in terms of p .

Sol. Given the middle number in the sequence of 5 consecutive even numbers $5p$.
 The other four numbers in the sequence in terms of p are $5p - 4$, $5p - 2$, $5p + 2$, $5p + 4$
 Hence, the other four numbers in sequence are p , $3p$, $7p$, and $9p$.

10. Write a 6-digit number that is divisible by 15, such that when the digits are reversed, it is divisible by 6.

Sol. We know that if the number is divisible by 3 and 5, then it is also divisible by 15.

Consider the number 643215.

Sum of the digits = $6 + 4 + 3 + 2 + 1 + 5 = 21$, which is divisible by 3.

Thus, 643215 is divisible by 3.

One's place = 5, it is also divisible by 5.

Hence, 643215 is divisible by 15.

One's place is not 0, because the digits are reversed, it becomes a 5-digit number.

Lakhs place is always taken as an even number.

Reversed the digits:

512346

One's place = 6, 512346 is divisible by 2.

Sum of the digits = $5 + 1 + 2 + 3 + 4 + 6 = 21$.

It is also divisible by 3.

Hence, 512346 is divisible by 6.

11. Deepak claims, "There are some multiples of 11 which, when doubled, are still multiples of 11. But other multiples of 11 don't remain multiples of 11 when doubled". Examine if his conjecture is true; explain your conclusion.

Sol. The multiples of 11 are: 11, 22, 33, 44, 55,...

When doubled, 22, 44, 66, 88, 110,.....

i.e. $(11) \times 2$, 11×4 , 11×6 , 11×8 , 11×10 ,.... are also multiples of 11.

False, if multiples of 11 are doubled, then the multiples of 11 are these numbers.

12. Determine whether the statements below are 'Always True', 'Sometimes True', or 'Never True'. Explain your reasoning.

(i) The product of a multiple of 6 and a multiple of 3 is a multiple of 9.

(ii) The sum of three consecutive even numbers will be divisible by 6.

(iii) If abcdef is a multiple of 6, then badcef will be a multiple of 6.

(iv) $8(7b - 3) - 4(11b + 1)$ is a multiple of 12.

Sol. (i) Always True,

The multiple of 6 can be written as $6a$, where a is an integer.

The multiple of 3 can be written as $3b$, where b is an integer.

$\therefore$ Product = $(6a) \times (3b) = 18(ab)$ is a multiple of 9.

(ii) Always True,

The sum of three consecutive even numbers will be divisible by 6.

For example $2 + 4 + 6 = 12$, $4 + 6 + 8 = 18$, $6 + 8 + 10 = 24$, $8 + 10 + 12 = 30$,...

These numbers are divisible by 6.

(iii) Always True, because one's place does not change.

(iv) Sometimes true,

Conclusion:

$8(7 \times 1 - 3) - 4(11 \times 1 + 1) = -16$, not divisible by 12.

$8(7 \times 10 - 3) - 4(4 \times 10 + 1) = 536 - 164 = 372$, divisible by 12.

13. Choose any 3 numbers. When is their sum divisible by 3? Explore all possible cases and generalise.

Sol. Let the three numbers be n_1, n_2 , and n_3 .
 Let their remainders when divided by 3 be r_1, r_2 , and r_3 .
 The sum $n_1 + n_2 + n_3$ is divisible by 3 if and only if $r_1 + r_2 + r_3$ is divisible by 3.
 Case 1: All remainders are 0.
 $r_1 = 0, r_2 = 0, r_3 = 0$
 Sum of remainders = $0 + 0 + 0 = 0$, which is divisible by 3.
 Case 2: All remainders are 1.
 $r_1 = 1, r_2 = 1, r_3 = 1$
 Sum of remainders = $1 + 1 + 1 = 3$, which is divisible by 3.
 Case 3: All remainders are 2.
 $r_1 = 2, r_2 = 2, r_3 = 2$
 Sum of remainders = $2 + 2 + 2 = 6$, which is divisible by 3.
 Case 4: One remainder is 0, one is 1, and one is 2.
 $r_1 = 0, r_2 = 1, r_3 = 2$ (in any order).
 Sum of remainders = $0 + 1 + 2 = 3$, which is divisible by 3.
 The sum of three numbers is divisible by 3 if and only if all three numbers have the same remainder when divided by 3, or if they all have different remainders when divided by 3.

14. Is the product of two consecutive integers always a multiple of 2? Why? What about the product of these consecutive integers? Is it always a multiple of 6? Why or why not? What can you say about the product of 4 consecutive integers? What about the product of five consecutive integers?

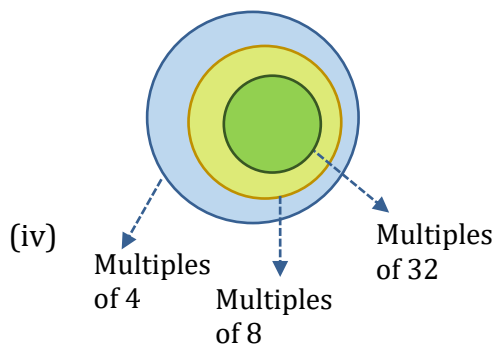
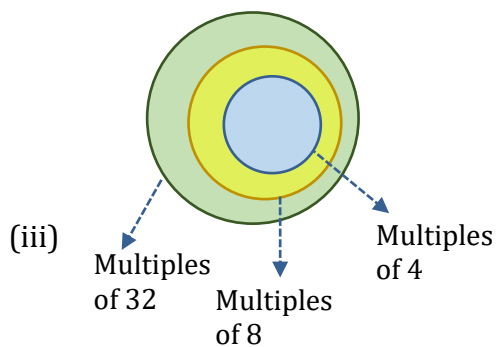
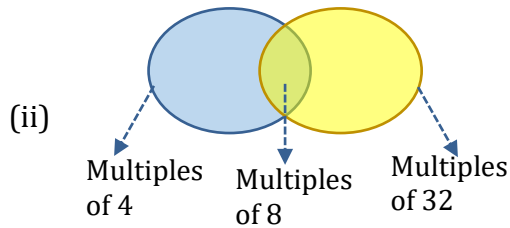
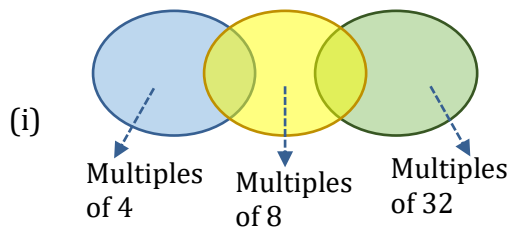
Sol. Yes, the product of two consecutive integers is always a multiple of 2.
 $1 \times 2 = 2, 2 \times 3 = 6, 5 \times 6 = 30, 10 \times 11 = 110$, and so on.
 Since we know that multiplying by an odd number and an even number is always an even number.
 No, it is not always a multiple of 6.
 $1 \times 2 = 2, 4 \times 5 = 20, 7 \times 8 = 56$
 Since it is not divisible by 6.
 The product of 4 consecutive integers
 $2 \times 3 \times 4 \times 5 = 120$,
 $4 \times 5 \times 6 \times 7 = 840$,
 $5 \times 6 \times 7 \times 8 = 1680$
 We can say that the product of 4 consecutive integers, divisible by 12.
 The product of five consecutive integers is:
 $1 \times 2 \times 3 \times 4 \times 5 = 120$,
 $2 \times 3 \times 4 \times 5 \times 6 = 720$,
 $3 \times 4 \times 5 \times 6 \times 7 = 2520$
 Hence, we can say that the product of five consecutive integers is always divisible by 24.

15. Solve the cryptarithms
 (i) $EF \times E = GGG$
 (ii) $WOW \times 5 = MEOW$

Sol. (i) This means a 2-digit number multiplied by 5 gives a 3-digit number.
 2-digit number = 20, 21, ..., 99
 $37 \times 3 = 111$, all conditions are satisfied.

(ii) This means a 3-digit number multiplied by 5 gives 4-digit numbers.
 Pick 3-digit number = 200, 201, ..., 999
 $525 \times 5 = 2625$

16. Which of the following Venn diagrams captures the relationship between the multiples of 4, 8, and 32?



Sol. (iv) Multiples of 4 are: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, ...
 Multiples of 8 are: 8, 16, 24, 32, 40, 48, 56, 64, ...
 Multiples of 32 are: 32, 64, 96, 128, ...
 The Venn diagram captures the relationship between the multiples of 4, 8, and 32:

