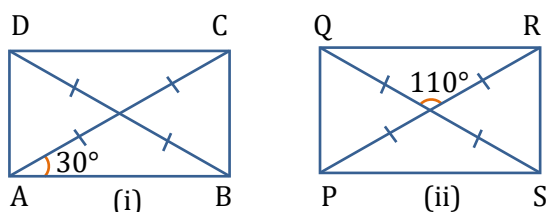


NCERT QUESTIONS WITH SOLUTIONS

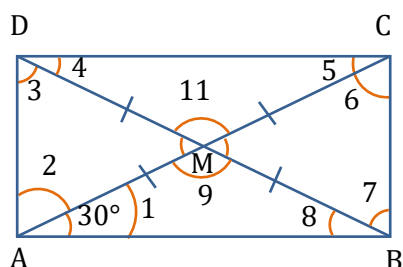
FIGURE IT OUT-01

1. Find all the other angles inside the following rectangles.



Solution

(i) The given rectangle is ABCD.



We have $\angle 1 = 30^\circ$

$$\angle 1 + \angle 2 = 90^\circ$$

$$\therefore \angle 2 = 90^\circ - \angle 1 = 90^\circ - 30^\circ = 60^\circ$$

$$MD = MA$$

$$\Rightarrow \angle 3 = \angle 2 = 60^\circ$$

$$\angle 3 + \angle 4 = 90^\circ$$

$$\therefore \angle 4 = 90^\circ - \angle 3 = 90^\circ - 60^\circ = 30^\circ$$

$$MC = MD$$

$$\Rightarrow \angle 5 = \angle 4 = 30^\circ$$

$$\angle 5 + \angle 6 = 90^\circ$$

$$\therefore \angle 6 = 90^\circ - \angle 5 = 90^\circ - 30^\circ = 60^\circ$$

$$MB = MC$$

$$\Rightarrow \angle 7 = \angle 6 = 60^\circ$$

$$MB = MA$$

$$\angle 8 = \angle 1 = 30^\circ$$

In $\triangle AMB$, we have

$$\angle 1 + \angle 9 + \angle 8 = 180^\circ$$

$$\therefore 30^\circ + \angle 9 + 30^\circ = 180^\circ$$

$$\therefore \angle 9 + 180^\circ - 60^\circ = 120^\circ$$

$$\angle 11 = \angle 9 = 120^\circ \text{ (Vertically opposite angles)}$$

$$\angle 9 + \angle 10 = 180^\circ \text{ (Linear angles)}$$

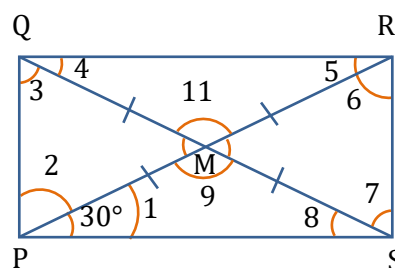
$$\therefore \angle 10 = 180^\circ - 120^\circ = 60^\circ$$

$$\angle 12 = \angle 10 = 60^\circ \text{ (Vertically opposite angles)}$$

$$\therefore \angle 2 = 60^\circ, \angle 3 = 60^\circ, \angle 4 = 30^\circ, \angle 5 = 30^\circ, \angle 6 = 60^\circ, \angle 7 = 60^\circ, \angle 8 = 30^\circ, \angle 9 = 120^\circ, \angle 10 = 60^\circ, \angle 11 = 120^\circ, \text{ and}$$

$$\angle 12 = 60^\circ,$$

(ii) The given rectangle is PSRQ.



We have $\angle 9 = 110^\circ$

$$\angle 11 + \angle 9 = 110^\circ \text{ (Vertically opposite angles)}$$

$$\therefore \angle 9 + \angle 10 = 180^\circ \text{ (Linear angles)}$$

$$MD = MA$$

$$\Rightarrow \angle 3 = \angle 2 = 60^\circ$$

$$\angle 3 + \angle 4 = 90^\circ$$

$$\therefore \angle 4 = 90^\circ - \angle 3 = 90^\circ - 60^\circ = 30^\circ$$

$$MC = MD$$

$$\Rightarrow \angle 5 = \angle 4 = 30^\circ$$

$$\angle 5 + \angle 6 = 90^\circ$$

$$\therefore \angle 6 = 90^\circ - \angle 5 = 90^\circ - 30^\circ = 60^\circ$$

$$MB = MC$$

$$\Rightarrow \angle 7 = \angle 6 = 60^\circ$$

$$MB = MA$$

$$\angle 8 = \angle 1 = 30^\circ$$

In $\triangle AMB$, we have

$$\angle 1 + \angle 9 + \angle 8 = 180^\circ$$

$$\therefore 30^\circ + \angle 9 + 30^\circ = 180^\circ$$

$$\therefore \angle 9 + 180^\circ - 60^\circ = 120^\circ$$

$\angle 11 = \angle 9 = 120^\circ$ (Vertically opposite angles)

$$\angle 9 + \angle 10 = 180^\circ \text{ (Linear angles)}$$

$$\therefore \angle 10 = 180^\circ - 120^\circ = 60^\circ$$

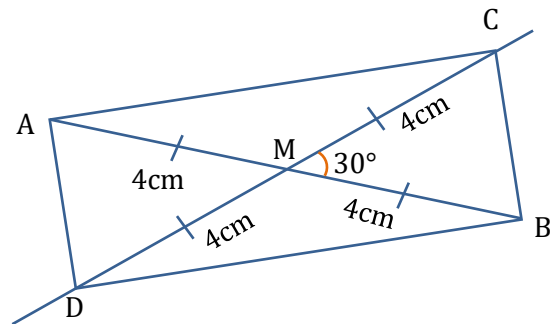
$\angle 12 = \angle 10 = 60^\circ$ (Vertically opposite angles)

$$\therefore \angle 2 = 60^\circ, \angle 3 = 60^\circ, \angle 4 = 30^\circ, \angle 5 = 30^\circ, \\ \angle 6 = 60^\circ, \angle 7 = 60^\circ, \angle 8 = 30^\circ, \angle 9 = 120^\circ, \\ \angle 10 = 60^\circ, \angle 11 = 120^\circ, \text{ and } \angle 12 = 60^\circ,$$

2. Draw a quadrilateral whose diagonals have equal lengths of 8 cm that bisect each other, and intersect at an angle of
(i) 30° (ii) 40° (iii) 90° (iv) 140°

Solution

- (i) Draw a line AB equal to 8 cm.
Take point M on AB such that $AM = BM = 4$ cm.
Using a protractor, draw an angle of 30° at M on MB.
On this line, take points C and D such that $MC = MD = 4$ cm.
Join AD, DB, BC, and CA.
ABCD is the required quadrilateral.



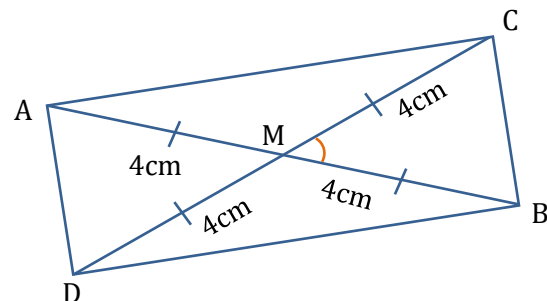
Since diagonals AB and CD are equal and are bisecting each other at M, ACBD is a rectangle.

- (ii) Draw a line AB equal to 8 cm.
Take point M on AB such that $AM = BM = 4$ cm.

Using a protractor, draw an angle of 40° at M on MB.

On this line, take points C and D such that $MC = MD = 4$ cm.

Join AD, DB, BC, and CA.
ABCD is the required quadrilateral.



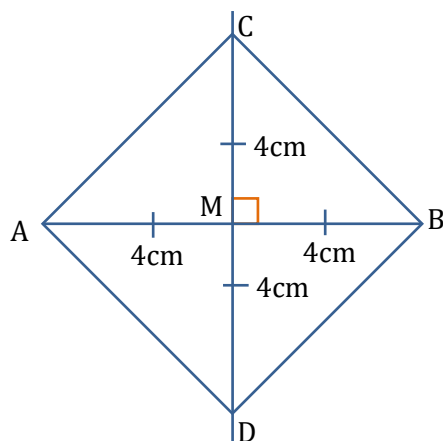
Since diagonals AB and CD are equal and are bisecting each other at M, ACBD is a rectangle.

- (iii) Draw a line AB equal to 8 cm.
Take a point M on AB such that $AM = BM = 4$ cm.

Using a protractor, draw an angle of 90° at M on MB.

On this line, take points C and D such that $MC = MD = 4$ cm.

Join AD, DB, BC, and CA. ACBD is the required square.



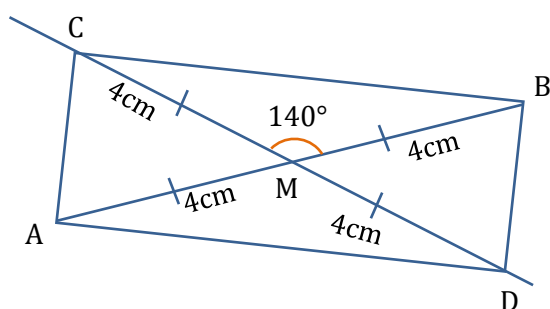
Since diagonals AB and CD are equal and are bisecting each other at M, and also the diagonals are perpendicular to each other, ACBD is a square.

- (iv) Draw a line AB equal to 8 cm. Take a point M on AB such that $AM = BM = 4$ cm.

Using a protractor, draw an angle of 140° at M on MB.

On this line, take points C and D such that $MC = MD = 4$ cm.

Join AD, DB, BC, and CA. ACBD is the required quadrilateral.



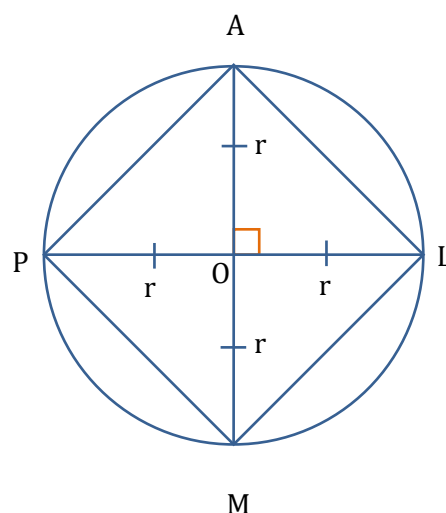
Since diagonals AB and CD are equal and are bisecting each other at M, ACBD is a rectangle.

3. Consider a circle with centre O. Line segments PL and AM are two perpendicular diameters of the circle.

What is the figure APML? Reason and/or experiment to figure this out.

Solution

In the figure, PL and AM are two perpendicular diameters of the circle. Let r be the radius of the circle.



Since $PL = PO + OL$

$$= r + r$$

$$= 2r$$

and $AM = AO + OM$

$$= r + r$$

$$= 2r$$

$$\therefore PL = AM$$

$\therefore$ In the quadrilateral APML, diagonals

PL and AM are equal and are perpendicular to each other.

Also, $OP = OA = OL = OM = r$

$\therefore$ Diameters PL and AM bisect each other at O.

$\therefore$ Quadrilateral APML is a square.

3. We have seen how to get 90° using paper folding. Now, suppose we do not have any paper but two sticks of equal length, and a thread. How do we make an exact 90° using these?

Solution

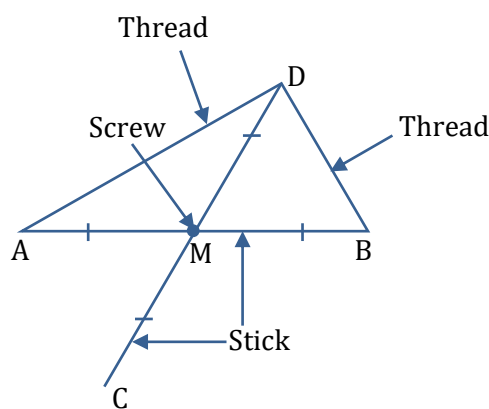
Let AB and CD be two sticks of equal length, say 6 cm.

Mark the midpoints of the sticks using a ruler.

Fix a screw to the sticks at their midpoints.

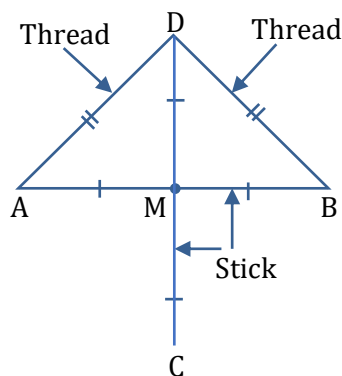
Using a thread, measure distances AD and BD.

Keep on moving the sticks about the screw, so that the distances AD and BD are equal.



In this position, fix the sticks by tightening the screw. The new positions of the sticks are shown in the figure.

Tie pieces of thread along AD and BD.



Consider $\triangle AMD$ and $\triangle BMD$. We have $AM = BM$, $AD = BD$ and MD is common

$\therefore$ By the SSS condition, $\triangle AMD$ and $\triangle BMD$ are congruent.

$\therefore \angle AMD = \angle BMD$

Also $\angle AMD + \angle BMD = 180^\circ$ (Linear angles)

$\therefore \angle AMD + \angle AMD = 180^\circ$

$\Rightarrow 2\angle AMD = 180^\circ$

$\Rightarrow \angle AMD = 90^\circ$

$\therefore \angle AMD = \angle BMD = 90^\circ$

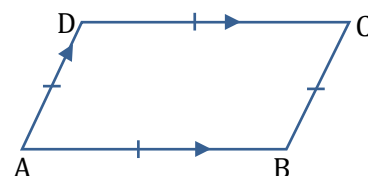
$\therefore$ Angle between the sticks is 90°

3. We saw that one of the properties of a rectangle is that its opposite sides are parallel. Can this be chosen as a definition of a rectangle? In other words, is every quadrilateral that has opposite sides parallel and equal, a rectangle?

Solution

Let ABCD be a quadrilateral in which opposite sides are parallel and equal. Here $AB \parallel DC$ and $AD \parallel BC$. Also, $AB = DC$ and $AD = BC$. In the quadrilateral ABCD, opposite sides are equal. For ABCD to be a rectangle, we require each angle to be 90° .

Given information $AB \parallel DC$ and $AD \parallel BC$ can not help us to prove that each angle of ABCD is 90° .



$\therefore$ ABCD may not be a rectangle.

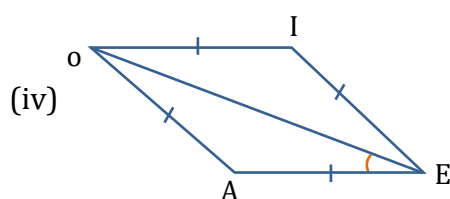
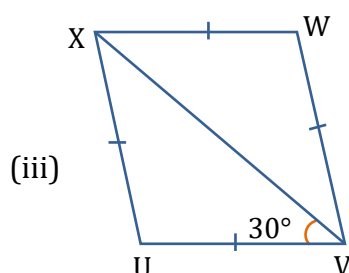
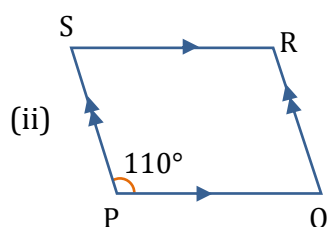
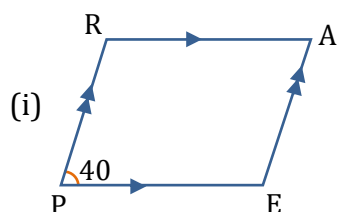
$\therefore$ A rectangle can not be defined as a

quadrilateral with equal and parallel opposite sides.

4.4 Quadrilaterals with Equal Sidelengths

Figure it out (Pg 102)

1. Find the remaining angles in the following quadrilaterals.



Solution

- (i) Since opposite sides of the quadrilateral PEAR are parallel, this is a parallelogram. In the given parallelogram, PE is a transversal to the parallel lines PR and EA.
- $\angle RPE$ and $\angle AEP$ are the internal angles on the same side of parallel lines.
- $$\therefore \angle RPE + \angle AEP = 180^\circ$$
- $$\Rightarrow 40^\circ + \angle AEP = 180^\circ$$

$$\Rightarrow \angle AEP = 180^\circ - 40^\circ = 140^\circ$$

We know that opposite angles of a parallelogram are equal.

$$\therefore \angle EAR = \angle EPR = 40^\circ \text{ and } \angle ARP = \angle AEP = 140^\circ.$$

- (ii) Since opposite sides of the quadrilateral PQRS are parallel, this is a parallelogram. In the given parallelogram, PQ is a transversal to the parallel lines PS and QR.

$\angle SPQ$ and $\angle RQP$ are the internal angles on the same sides of the parallel lines.

$$\therefore \angle SPQ + \angle RQP = 180^\circ$$

$$\Rightarrow 110^\circ + \angle RQP = 180^\circ$$

$$\Rightarrow \angle RQP = 180^\circ - 110^\circ = 70^\circ$$

We know that the opposite angles of a parallelogram are equal.

$$\therefore \angle QRS = \angle QPS = 110^\circ \text{ and } \angle PSR = \angle RQP = 70^\circ.$$

- (iii) Here, quadrilateral UVWX is a rhombus, because all sides are equal. We have $\angle 1 = 30^\circ$.

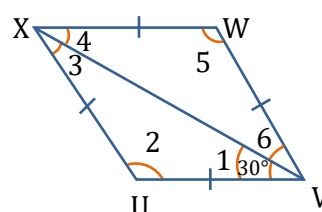
In a rhombus, diagonals bisect the angles of the rhombus.

$$\therefore \angle 6 = 30^\circ$$

In $\triangle UVX$, $\angle 1 = \angle 3$ ($\because UV = UX$)

$$\therefore \angle 3 = 30^\circ$$

$$\therefore \angle 4 = 30^\circ$$



- In $\triangle UVX$, $\angle 1 + \angle 2 + \angle 3 = 180^\circ$.
- $$\therefore 30^\circ + \angle 2 + 30^\circ = 180^\circ$$
- $$\Rightarrow \angle 2 = 180^\circ - 60^\circ = 120^\circ$$
- $$\Rightarrow \angle 5 = \angle 2 = 120^\circ \text{ } (\because \text{Opposite angles are equal})$$

equal)

$\therefore \angle 2 = 120^\circ, \angle 3 = 30^\circ, \angle 4 = 30^\circ, \angle 5 = 120^\circ$
and $\angle 6 = 30^\circ$.

(iv)

Here, quadrilateral UVWX is a rhombus, because all sides are equal.

We have $\angle 1 = 20^\circ$.

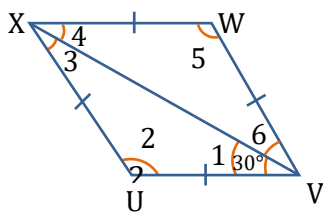
In a rhombus, diagonals bisect the angles of the rhombus.

$\therefore \angle 6 = 20^\circ$

In $\triangle UVX$, $\angle 1 = \angle 3$ ($\because UV = UX$)

$\therefore \angle 3 = 20^\circ$

$\therefore \angle 4 = 20^\circ$



In $\triangle UVX$, $\angle 1 + \angle 2 + \angle 3 = 180^\circ$.

$\therefore 20^\circ + \angle 2 + 20^\circ = 180^\circ$

$\Rightarrow \angle 2 = 180^\circ - 40^\circ = 140^\circ$

$\Rightarrow \angle 5 = \angle 2 = 140^\circ$ ($\because$ Opposite angles are equal)

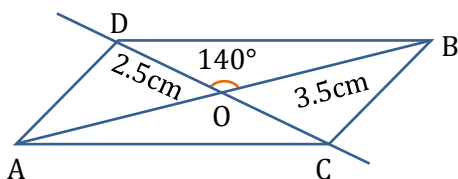
$\therefore \angle 2 = 140^\circ, \angle 3 = 40^\circ, \angle 4 = 40^\circ, \angle 5 = 120^\circ$
and $\angle 6 = 40^\circ$.

2. Using the diagonal properties, construct a parallelogram whose diagonals are of lengths 7 cm and 5 cm, and intersect at an angle of 140° .

Solution

Draw a line AB equal to 7 cm.

Take point O on AB such that $AO = OB = 3.5$ cm.



On OB, draw an angle of 140° at O.

Take points C and D on the line of angle so that $OC = OD = 2.5$ cm.

$\therefore CD = 5$ cm, and O is the midpoint of AB and AC.

Join AC, CB, BD, and DA.

ACBD is a quadrilateral, and its diagonals AB and CD bisect at O.

$\therefore$ ACBD is the required parallelogram.

3. Using the diagonal properties, construct a rhombus whose diagonals are of lengths 4 cm and 5 cm.

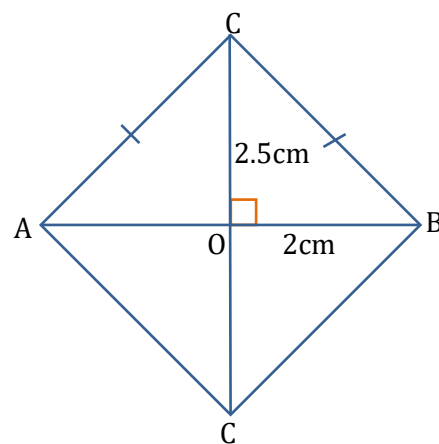
Solution

Draw a line AB equal to 4 cm.

Take a point O on AB such that $AO = OB = 2$ cm.

On AB, draw a line perpendicular to it and passing through O.

Take points C and D on this perpendicular so that $OC = OD = 2.5$ cm.



$\therefore CD = 5$ cm, and O is the midpoint of AB and CD.

Join AC, CB, BD, and DA.

ACBD is a quadrilateral, and its diagonals AB and CD are bisecting at O and are also perpendicular to each other.

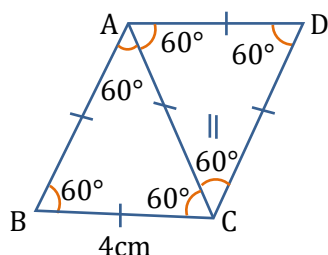
$\therefore$ ACBD is the required rhombus.

Figure it out (Pg 107)

- Find all the sides and the angles of the quadrilateral obtained by joining two equilateral triangles with sides 4 cm.

Solution

Let two equilateral triangles of sides 4 cm be joined as shown below.



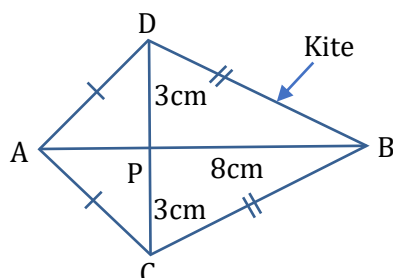
The sides of the quadrilateral ABCD are 4 cm each.

The angles of the quadrilateral ABCD are $\angle A = 60^\circ + 60^\circ = 120^\circ$, $\angle B = 60^\circ$, $\angle C = 60^\circ + 60^\circ = 120^\circ$ and $\angle D = 60^\circ$.

- Construct a kite whose diagonals are of lengths 6 cm and 8 cm.

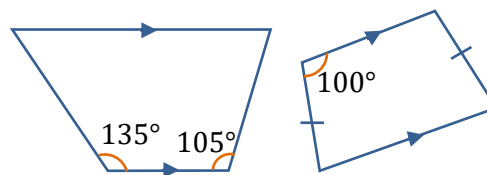
Solution

Draw a line AB equal to 8 cm. Take a point P on the line AB. Draw a perpendicular line to AB passing through P. Take points C and D on this perpendicular such that $PC = PD = 3$ cm. Join AC, CB, BD, and DA.



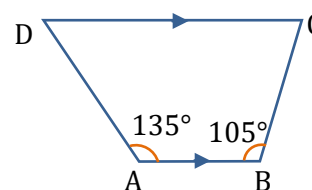
ACBD is the required kite with diagonals of lengths 6 cm and 8 cm.

- Find the remaining angles in the following trapeziums—



Solution

- Let the given trapezium be ABCD. Lines AB and DC are Parallel



$$\therefore \angle A + \angle D = 180^\circ \text{ and } \angle B + \angle C = 180^\circ.$$

$$\therefore \angle A + \angle D = 180^\circ$$

$$\Rightarrow 135^\circ + \angle D = 180^\circ$$

$$\Rightarrow \angle D = 180^\circ - 135^\circ = 45^\circ.$$

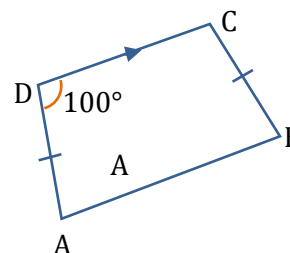
$$\therefore \angle B + \angle C = 180^\circ$$

$$\Rightarrow 105^\circ + \angle C = 180^\circ$$

$$\Rightarrow \angle C = 180^\circ - 105^\circ = 75^\circ.$$

$\therefore$ The remaining angles are 45° and 75° .

- Let the given trapezium be ABCD. Since $AD = BC$, ABCD is an isosceles trapezium.



$\therefore$ Angles opposite to the equal sides are equal.

$$\therefore \angle C = \angle D = 100^\circ$$

Lines AB and DC are parallel.

$$\therefore \angle A + \angle D = 180^\circ \text{ and } \angle B + \angle C = 180^\circ$$

$$\therefore \angle A + \angle D = 180^\circ$$

$$\Rightarrow \angle A + 100^\circ = 180^\circ$$

$$\Rightarrow \angle A = 180^\circ - 100^\circ$$

$$\Rightarrow \angle A = 80^\circ$$

$$\therefore \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle B + 100^\circ = 180^\circ$$

$$\Rightarrow \angle B = 180^\circ - 100^\circ$$

$$\Rightarrow \angle B = 80^\circ$$

$\therefore$ Remaining angles are $\angle A = 80^\circ$, $\angle B = 80^\circ$ and $\angle C = 100^\circ$.

4. Draw a Venn diagram showing the set of parallelograms, kites, rhombuses, rectangles, and squares. Then, answer the following questions—

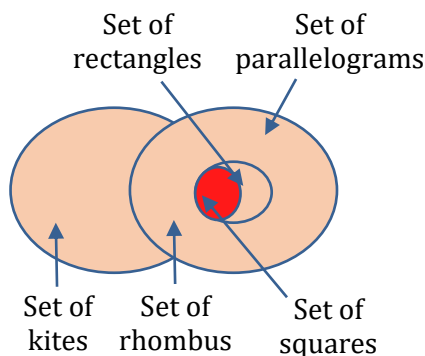
- What is the quadrilateral that is both a kite and a parallelogram?
- Can there be a quadrilateral that is both a kite and a rectangle?
- Is every kite a rhombus? If not, what is the correct relationship between these two types of quadrilaterals?

Solution

We know the following:

- Every rectangle is a parallelogram.
- Every square is a rectangle.
- Every square is a rhombus.
- Every rhombus is a kite.

The following Venn diagram shows the sets of parallelograms, kites, rhombuses, rectangles, and squares



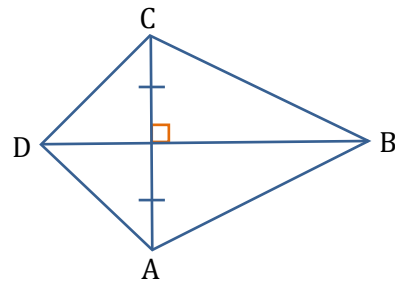
(i) The set of rhombuses is common to both the set of kites and the set of parallelograms.
 $\therefore$ A rhombus is both a kite and a parallelogram.

(ii) A kite is not a rectangle, and a rectangle is not a kite.

$\therefore$ There can be no quadrilateral that is both a kite and a rectangle.

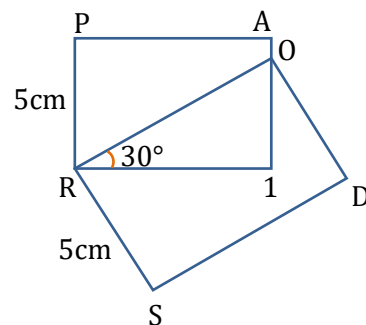
Also, there is no common portion of the set of kites and the set of rectangles.

(iii) Every kite is not a rhombus. In the given figure, the kite ABCD is not a rhombus.



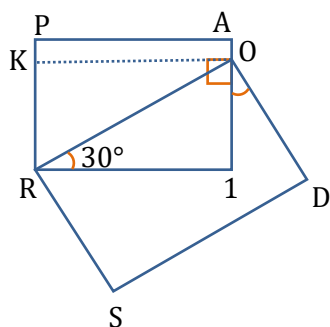
The correct relationship is that every rhombus is a kite.

5. If PAIR and RODS are two rectangles, find $\angle IOD$.



Solution

From O, draw a line OK parallel to RI.



$\therefore \angle KOR = \angle ORI = 30^\circ$ (Alternate angles)

$\therefore \angle KOR + \angle ROI = 90^\circ$

$\Rightarrow 30^\circ + \angle ROI = 90^\circ$

$\Rightarrow \angle ROI = 90^\circ - 30^\circ$

$\Rightarrow \angle ROI = 60^\circ$

Also, $\angle ROI + \angle IOD = 90^\circ$

$\Rightarrow 60^\circ + \angle IOD = 90^\circ$

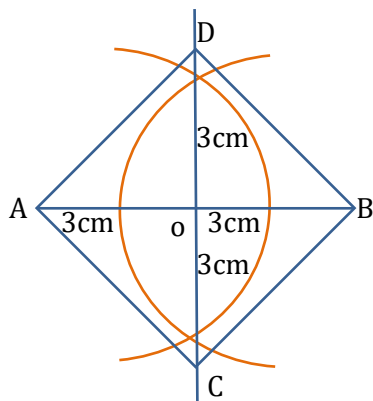
$\Rightarrow \angle IOD = 90^\circ - 60^\circ$

$\Rightarrow \angle IOD = 30^\circ$.

6. Construct a square with diagonal 6 cm without using a protractor.

Solution

Draw a line AB equal to 6 cm.



We have $6 \div 2 = 3$.

With centre at A and B, draw arcs of radius slightly greater than 3 cm, say, 4 cm.

Join the points of intersection of the arcs.

Let this line intersect AB at O.

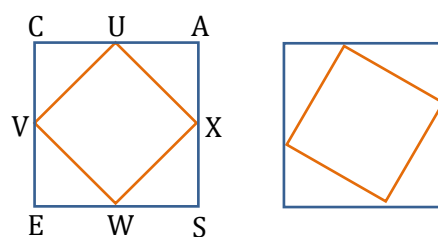
Take points C and D on the perpendicular line so that $OC = OD = 3$

cm.

Join AC, CB, BD, and DA.

ACBD is the required square with diagonals equal to 6 cm.

7. CASE is a square. The points U, V, W and X are the midpoints of the sides of the square. What type of quadrilateral is UVWX? Find this by using geometric reasoning, as well as by construction and measurement. Find other ways of constructing a square within a square such that the vertices of the inner square lie on the sides of the outer square, as shown in Figure (b).



(a)

(b)

Solution

(a) U, V, W, and X are the midpoints of the sides of the square.

In $\triangle VCU$ and $\triangle UAX$,

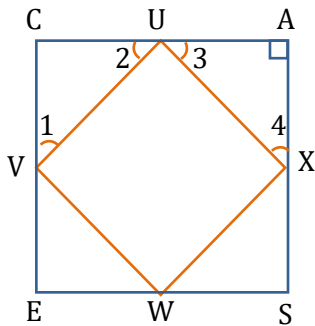
we have $VC = UA$, $\angle VCU = \angle UAX = 90^\circ$, and $CU = AX$.

$\therefore$ By the SAS condition, $\triangle VCU$ and $\triangle UAX$ are congruent.

$\therefore VU = UX$

Similarly, $VU = XW$, $VU = WV$.

$\therefore$ Sides of the quadrilateral UVWX are equal.



In $\triangle VCU$, $VC = CU$

$$\Rightarrow \angle 1 = \angle 2$$

$$\text{Also, } \angle 1 + \angle C + \angle 2 = 180^\circ$$

$$\Rightarrow \angle 1 + 90^\circ + \angle 1 = 180^\circ$$

$$\Rightarrow 2\angle 1 = 90^\circ$$

$$\Rightarrow \angle 1 = 45^\circ$$

$$\therefore \angle 2 \text{ is also } 45^\circ.$$

$$\text{Similarly, } \angle 3 = \angle 4 = 45^\circ$$

$$\text{We have } \angle 2 + \angle VUX + \angle 3 = 180^\circ$$

$$\Rightarrow 45^\circ + \angle VUX + 45^\circ = 180^\circ$$

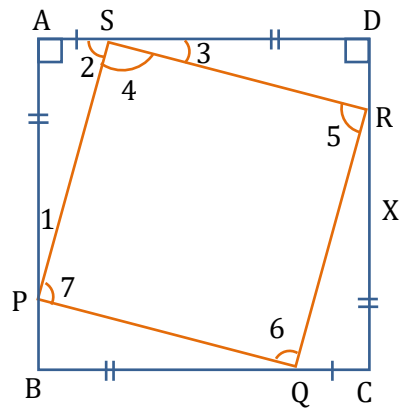
$$\Rightarrow \angle VUX = 180^\circ - 90^\circ$$

$$\Rightarrow \angle VUX = 90^\circ$$

$$\text{Similarly, } \angle VXW = 90^\circ, \angle XWV = 90^\circ \text{ and } \angle WVU = 90^\circ.$$

$\therefore$ By definition, the quadrilateral UVWX is a square.

- (b) Let ABCD be a square. Take points P, Q, R, and S such that $AS = BP = CQ = DR$.



Since the sides of squares are equal, we have $DS = AP = BQ = CR$.

In $\triangle PAS$ and $\triangle SDR$, we have

$$PA = SD, \angle PAS = \angle SDR = 90^\circ, \text{ and } AS = DR.$$

$\therefore$ By the SAS condition, $\triangle PAS$ and $\triangle SDR$ are congruent.

$$\therefore PS = SR$$

$$\text{Similarly, } PS = RQ, PS = QP.$$

$\therefore$ Sides of the quadrilateral PQRS are equal.

$$\text{In } \triangle PAS, \angle 1 + \angle 2 + 90^\circ = 180^\circ$$

$$\Rightarrow \angle 1 + \angle 2 = 90^\circ$$

$$\Rightarrow \angle 3 + \angle 2 = 90^\circ (\because \angle 1 = \angle 3)$$

$$\text{Also, } \angle 2 + \angle 4 + \angle 3 = 180^\circ$$

$$\Rightarrow 90^\circ + \angle 4 = 180^\circ$$

$$\Rightarrow \angle 4 = 180^\circ - 90^\circ$$

$$\Rightarrow \angle 4 = 90^\circ$$

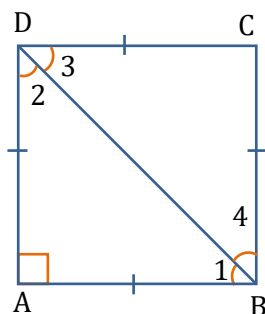
$$\therefore \text{Similarly, } \angle 5 = 90^\circ, \angle 6 = 90^\circ, \text{ and } \angle 7 = 90^\circ.$$

By definition, the quadrilateral PQRS is a square.

8. If a quadrilateral has four equal sides and one angle of 90° , will it be a square? Find the answer using geometric reasoning as well as by construction and measurement.

Solution

Let ABCD be a quadrilateral such that $AB = BC = CD = DA$ and $\angle DAB = 90^\circ$. Join BD.



In $\triangle ADB$ and $\triangle CDB$, we have
 $AD = CD$, $AB = CB$, and DB is a common side.

$\therefore \triangle ADB$ and $\triangle CDB$ are congruent.

$\therefore \angle C = \angle A = 90^\circ$

In $\triangle DAB$, $\angle 1 = \angle 2$ ($\because AB = AD$)

Also, $\angle 1 + 90^\circ + \angle 2 = 180^\circ$

$\Rightarrow \angle 1 + \angle 2 = 90^\circ$

$\Rightarrow \angle 1 = 45^\circ$ and $\angle 2 = 45^\circ$ ($\because \angle 1 = \angle 2$)

In $\triangle CDB$, $\angle 3 = \angle 4$ ($\because CD = CB$)

Also, $\angle 3 + 90^\circ + \angle 4 = 180^\circ$

$\Rightarrow \angle 3 + \angle 4 = 90^\circ$

$\Rightarrow \angle 3 = \angle 4 = 45^\circ$ ($\because \angle 3 = \angle 4$)

$\therefore \angle ABC = \angle 1 + \angle 4 = 45^\circ + 45^\circ = 90^\circ$

and $\angle ADC = \angle 2 + \angle 3 = 45^\circ + 45^\circ = 90^\circ$.

$\therefore$ Each angle of the quadrilateral $ABCD$ is 90° .

$\therefore ABCD$ is a square.

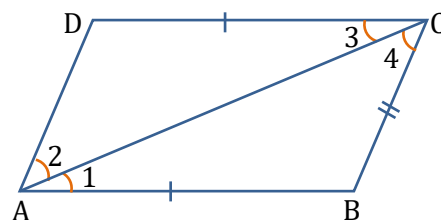
Also, by measurement, we find $AB = BC = CD = DA$ and $\angle A = \angle B = \angle C = \angle D = 90^\circ$.

9. What type of a quadrilateral is one in which the opposite sides are equal? Justify your answer.

Hint: Draw a diagonal and check for congruent triangles.

Solution

Let $ABCD$ be a quadrilateral in which opposite sides are equal.
 Join AC .



In $\triangle ADC$ and $\triangle CBA$, we have

$AD = CB$, $DC = BA$, and AC is common.

$\therefore$ By the SSS condition, $\triangle ADC$ and $\triangle CBA$ are congruent.

$\therefore \angle 1 = \angle 3$ and $\angle 2 = \angle 4$

AC is a transversal of lines AB and DC , and alternate angles $\angle 1$ and $\angle 3$ are equal.

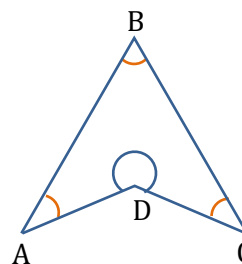
$\therefore$ Lines AB and DC are parallel.

AC is a transversal of lines AD and BC , and alternate angles $\angle 2$ and $\angle 4$ are equal.

$\therefore$ Lines AD and BC are parallel.

$\therefore$ By definition, the quadrilateral $ABCD$ is a parallelogram.

10. Will the sum of the angles in a quadrilateral such as the following one also be 360° ? Find the answer using geometric reasoning as well as by constructing this figure and measuring.



Solution

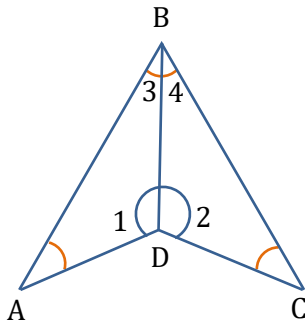
In the given quadrilateral, join BD .

In $\triangle ABD$, we have

$\angle A + \angle 3 + \angle 1 = 180^\circ$

In $\triangle CBD$, we have

$\angle C + \angle 4 + \angle 2 = 180^\circ$



Adding, we get

$$(\angle A + \angle 3 + \angle 1) + (\angle C + \angle 4 + \angle 2) = 180^\circ + 180^\circ$$

$$\Rightarrow \angle A + (\angle 3 + \angle 4) + \angle C + (\angle 1 + \angle 2) = 360^\circ$$

$$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360^\circ$$

$\therefore$ The sum of the angles of the quadrilateral ABCD is 360° .

Also, by using a protractor, we find that the sum of all angles is 360° .

11. State whether the following statements are true or false. Justify your answers.

(i) A quadrilateral whose diagonals are equal and bisect each other must be a square.

(ii) A quadrilateral having three right angles must be a rectangle.

(iii) A quadrilateral whose diagonals bisect each other must be a parallelogram.

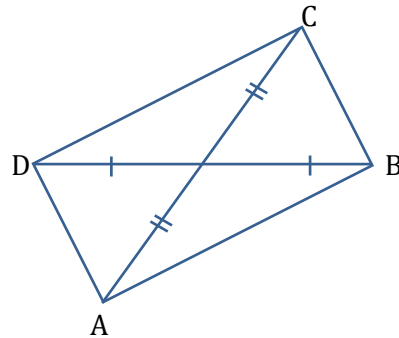
(iv) A quadrilateral whose diagonals are perpendicular to each other must be a rhombus.

(v) A quadrilateral in which the opposite angles are equal must be a parallelogram. (vi) A quadrilateral in which all the angles are equal is a rectangle.

(vii) Isosceles trapeziums are parallelograms.

Solution

(i) A quadrilateral whose diagonals are equal and bisect each other need not be a square. In the figure, diagonals AC and DB are equal and bisect each other. Such a quadrilateral is always a rectangle.



$\therefore$ The given statement is false.

(ii) Let ABCD be a quadrilateral having three right angles at A, D, and C.

$$\text{We have } \angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\Rightarrow 90^\circ + \angle B + 90^\circ + 90^\circ = 360^\circ$$

$$\Rightarrow \angle B = 360^\circ - 270^\circ$$

$$\Rightarrow \angle B = 90^\circ$$

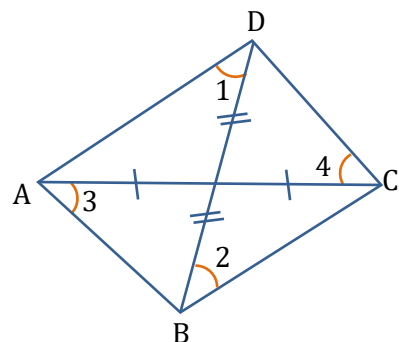
$\therefore$ Each angle of ABCD is 90° .

$\therefore$ Given quadrilateral is a rectangle.



$\therefore$ The given statement is true

(iii) In the quadrilateral ABCD, the diagonals AC and BD bisect each other. Here, $\triangle AOD$ and $\triangle COB$ are congruent. $\therefore \angle 1 = \angle 2$



$\therefore BC$ is parallel to AD .

Also, $\triangle AOB$ and $\triangle COD$ are congruent.

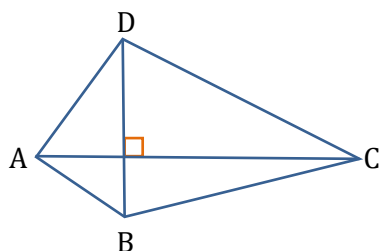
$\therefore \angle 3 = \angle 4$

$\therefore AB$ is parallel to DC .

Since opposite sides of $ABCD$ are parallel, it must be a parallelogram.

$\therefore$ The given statement is true.

- (iv) Let $ABCD$ be a quadrilateral whose diagonals AC and BD are perpendicular to each other.



This quadrilateral may not be a rhombus, because the diagonals AC and BC may not bisect each other.

$\therefore$ The given statement is false.

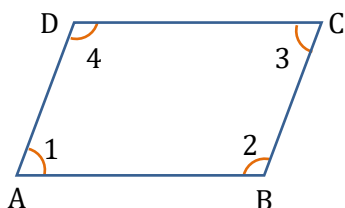
- (v) Let $ABCD$ be a quadrilateral in which $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$.

We have, $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360^\circ$.

$$\Rightarrow \angle 1 + \angle 2 + \angle 1 + \angle 2 = 360^\circ$$

$$\Rightarrow 2(\angle 1 + \angle 2) = 360^\circ$$

$$\Rightarrow \angle 1 + \angle 2 = 180^\circ$$



AB is a transversal of lines AD and BC , and the sum of internal angles $\angle 1$ and $\angle 2$ on the same side is 180° .

$\therefore$ Lines AD and BC are parallel.

Again, $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360^\circ$

$$\Rightarrow \angle 3 + \angle 2 + \angle 3 + \angle 2 = 360^\circ$$

$$\Rightarrow 2(\angle 2 + \angle 3) = 360^\circ$$

$$\Rightarrow \angle 2 + \angle 3 = 180^\circ$$

BC is a transversal of lines AB and DC , and the sum of internal angles $\angle 2$ and $\angle 3$ on the

same side is 180° .

$\therefore$ Lines AB and DC are parallel.

$\therefore$ Opposite sides of quadrilateral $ABCD$ are parallel.

$\therefore ABCD$ is a parallelogram.

$\therefore$ The given statement is true.

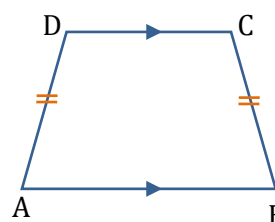
- (vi) Let $ABCD$ be a quadrilateral, where $\angle 1, \angle 2, \angle 3$, and $\angle 4$ are all equal.

We have $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360^\circ$

$$\therefore \angle 1 + \angle 1 + \angle 1 + \angle 1 = 360^\circ$$

$$\Rightarrow 4\angle 1 = 360^\circ$$

$$\Rightarrow \angle 1 = 90^\circ$$



$$\therefore \angle 2 = 90^\circ, \angle 3 = 90^\circ, \angle 4 = 90^\circ$$

We have, $\angle 5 + \angle 6 = 90^\circ$

and $\angle 6 + 90^\circ + \angle 8 = 180^\circ$

$$\Rightarrow \angle 5 + \angle 6 = \angle 6 + \angle 8$$

$$\Rightarrow \angle 5 = \angle 8$$

Also, $\angle 7 + 90^\circ + \angle 5 = 180^\circ$

$$\Rightarrow \angle 7 + \angle 5 = 90^\circ$$

$$\Rightarrow \angle 5 + \angle 6 = \angle 7 + \angle 5$$

$$\Rightarrow \angle 6 = \angle 7$$

In $\triangle DAB$ and $\triangle BCD$, we have $\angle 5 = \angle 8, \angle 7 = \angle 6$, and side BD is common.

$\therefore$ By the ASA condition, $\triangle DAB$ and $\triangle BCD$ are congruent.

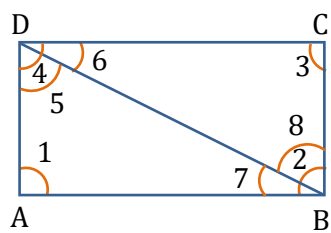
$\therefore DA = BC$ and $AB = CD$

$\therefore$ Opposite sides of $ABCD$ are equal.

$\therefore ABCD$ is a rectangle.

$\therefore$ The given statement is true

- (vii) An isosceles trapezium $ABCD$ can not be a parallelogram because it has two non-parallel equal lines AD and BC .



∴ The given statement is false.