# NCERT QUESTIONS WITH SOLUTIONS

## FIGURE IT OUT-01

**1.** Suppose you are using the number system that uses sticks to represent numbers, as in Method 1. Without using either the number names or the numerals of the Hindu number system, give a method for adding, subtracting, multiplying and dividing two numbers or two collections of sticks.

**Sol.** **Addition (Join the sticks):**

Just put both collections of sticks together.

You have:

|| (2 sticks) and ||| (3 sticks)

Join them: || + ||| → |||||

You now have 5 sticks.

**Subtraction (Remove sticks):**

Remove sticks from a collection.

You have:

|||||| (6 sticks) and ||| (3 sticks)

Take away 3 sticks from 6:

|||||| − ||| → |||

You are left with 3 sticks.

**Multiplication (Make equal groups):**

Repeated addition – make that many groups of sticks and combine.

You want:

|| (2 sticks) × ||| (3 sticks)

That means: 3 groups of 2 sticks

Group 1: ||

Group 2: ||

Group 3: ||

Put all sticks together:

|| + || + || → |||||| (You have 6 sticks.)

**Division (Make equal groups from a set):**

Divide sticks into equal groups.

You have:

|||||| (6 sticks) and ||| (3 sticks)

Take away 3 sticks from 6:

|||||| ÷ ||| → || (group size = 2)

How many groups of 2 can you make?

Group 1: ||

Group 2: ||

Group 3: ||

You made 3 groups of 2 sticks.

So, $6 \div 2 = 3$ (but again, we don't say "3" we just show 3 groups of sticks.)

**2.** One way of extending the number system in Method 2 is by using strings with more than one letter — for example, we could use 'aa' for 27. How can you extend this system to represent all the numbers? There are many ways of doing it!

**Sol.** a, b, c, ..., z are 26 numbers

aa, bb, ..., zz are 26 more numbers.

i.e., aa = 27; bb = 28

aaa, bbb, .......zzz are 26 more numbers

i.e., aaa = 53, bbb = 54

**3.** Try making your own number system.

**Sol.** Let base be 3

Then $3^0 = 1 = A$, $3^1 = 3 = B$, $3^2 = 9 = C$....,

| | | |
|---|---|---|
| 1 | A | $3^0 = 1$ |
| 2 | AA | 1+1 |
| 3 | B | $3^1=3$ |
| 4 | BA | 3+1 |
| 5 | BAA | 3+1+1 |
| 6 | BB | 3+3 |
| 7 | BBA | 3+3+1 |
| 8 | BBAA | 3+3+1+1 |
| 9 | C | $3^2 =9$ |
| 10 | CA | 9+1 |
| 11 | CAA | 9+1+1 |
| 12 | CB | 9+3 |

## FIGURE IT OUT-02

**1.** Represent the following numbers in the Roman system.

(i) 1222 (ii) 2999

(iii) 302 (iv) 715

**Sol.** (i) 1222 = 1000 + 100 + 100 + 10 + 10 + 1 + 1 = MCCXXII

(ii) 2999 = 1000 + 1000 + (1000 – 100) + (100 – 10) + (10 – 1) = MMCMXCIX

(iii) 302 = 100 + 100 + 100 + 1 + 1 = CCCII

(iv) 715 = 500 + 100 + 100 + 10 + 5 = DCCXV

## FIGURE IT OUT-03

**1.** A group of indigenous people in a Pacific island use different sequences of number names to count different objects. Why do you think they do this?

**Sol.** Counting in twos is more efficient in representing numbers than, for example, a tally system.

**2.** Consider the extension of the Gumulgal number system beyond 6 in the same way of counting by 2s. Come up with ways of performing the different arithmetic operations (+, –, ×, ÷) for numbers occurring in this system, without using Hindu numerals. Use this to evaluate the following:

(i) (ukasar-ukasar-ukasar-ukasar-urapon) + (ukasar-ukasar-ukasar-urapon)

(ii) (ukasar-ukasar-ukasar-ukasar-urapon) – (ukasar-ukasar-ukasar)

(iii) (ukasar-ukasar-ukasar-ukasar-urapon) × (ukasar-ukasar)

(iv) (ukasar-ukasar-ukasar-ukasar-ukasar-ukasar-ukasar-ukasar) ÷ (ukasar-ukasar)

**Sol.** (i) uk - uk - uk - uk - ur

\+ uk - uk - uk - ur

uk - uk - uk - uk - uk - uk - uk - uk

[∵ ur + ur = uk]

(ii) uk - uk - uk - uk - ur

– uk - uk – uk

uk – ur

(iii) (uk–uk–uk–uk–uk–ur)×(uk–uk)

=(uk–uk–uk–uk–uk–ur)×uk + (uk–uk–uk–uk–uk–ur)×uk

=uk–uk–uk–uk–uk–uk–uk–uk–uk

+uk–uk–uk–uk–uk–uk–uk–uk–uk

= (uk–uk–.........uk) (18 times)

(iv) (uk–uk–uk–uk–uk–uk–uk–uk)÷(uk–uk)

=[(uk–uk)–(uk–uk)–(uk–uk)–(uk–uk)]÷(uk–uk)

= 4×[(uk–uk)÷(uk–uk)]

= ur - ur - ur - ur

= uk–uk

**3.** Identify the features of the Hindu number system that make it efficient when compared to the Roman number system.

**Sol.** (i) Hindu numbers have '0' and a place value system, which Roman numerals do not have.

(ii) The Hindu number system is a positional system, whereas the Roman system is not.

**4.** Using the ideas discussed in this section, try refining the number system you might have made earlier.

**Sol.** Try it yourself.

## FIGURE IT OUT-04

**1.** Represent the following numbers in the Egyptian system: 10458, 1023, 2660, 784, 1111, 70707.

**Sol.** 10458 = 10000 + 400 + 50 + 8

= 

1023 = 1000 + 20 + 3

= 

2660 = 2000 + 600 + 60

= 

784 = 700 + 80 + 4

= 

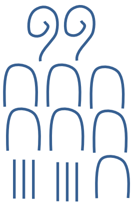
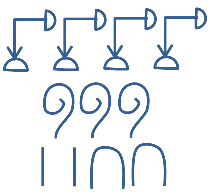
1111 = 1000 + 100 + 10 + 1

= 

70707 = 70000 + 700 + 7

= 

**2.** What numbers do these numerals stand for?

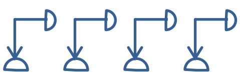
(i)  (ii) 

**Sol.** (i)  = 100 × 2 = 200

 = 10 × 7 = 70

 = 1 × 6 = 6

⇒ 200 + 70 + 6 = 276

(ii)  = 1000 × 4 = 4000

 = 100 × 3 = 300

 = 10 × 2 = 20

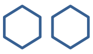
 = 1 × 2 = 2

⇒ 4000 + 300 + 20 + 2 = 4322

## FIGURE IT OUT-05

**1.** Write the following numbers in the above base-5 system using the symbols in Table-2: 15, 50, 137, 293, 651.

**Sol.** 15 = 5 + 5 + 5 = 

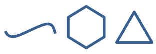
50 = 25 + 25 = 

137 = 125 + 5 + 5 + 1 + 1

= 

293 = 125+125+25 + 5 + 5 + 5 + 1 + 1 + 1

= 

651 = 625 + 25 + 1 = 

**2.** Is there a number that cannot be represented in our base-5 system above? Why or why not?

**Sol.** Zero (0) cannot be represented in our base-5 system as there is no symbol for it.

**3.** Compute the landmark numbers of a base-7 system. In general, what are the landmark numbers of a base-n system?

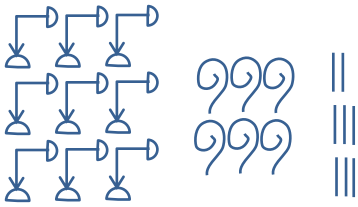
**Sol.** $7^0 = 1$, $7^1 = 7$, $7^2 = 49$, $7^3 = 343$, $7^4 = 2401$

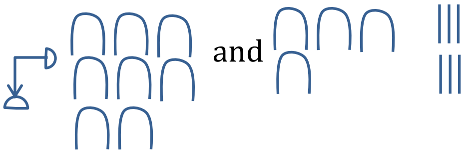
Hence, 1, 7, 49, 343, 2401 are landmark numbers of base 7.

The landmark numbers of a base-n number system are the powers of n starting from $n^0 = 1, n, n^2, n^3, \ldots$

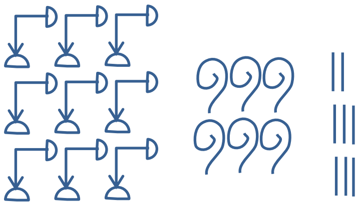
## FIGURE IT OUT-06

**1.** Add the following Egyptian numerals:

(i)  and

(ii) 

**Sol.** (i) Here,



= 9 × 1000 + 6 × 100 + 8 × 1

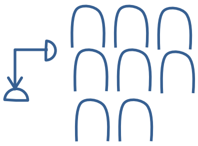
= 9000 + 600 + 8 = 9608

And 

= 5 × 100 + 7 × 1 = 500 + 7 = 507

So, 9608 + 507

= 10115 or 

(ii) Here, 

= 1000 + 8 × 10 = 1080

And 

= 4 × 10 + 6 × 1 = 40 + 6 = 46

So, 1080 + 46 = 1126

Or, 

**2.** Add the following numerals that are in the base-5 system that we created:

○⬡⬡□△△ + ○○○⬡□□△△

Remember that in this system, 5 times a landmark number gives the next one!

**Sol.**

  ○⬡⬡□△△

+ ○○○⬡□□△△

―――――――――――――

○○○○⬡⬡⬡□□□△△△△

## FIGURE IT OUT-07

**1.** Can there be a number whose representation in Egyptian numerals has one of the symbols occurring 10 or more times? Why not?

**Sol.** No, ten times any landmark number will give the next landmark number.

**2.** Create your own number system of base 4, and represent numbers from 1 to 16.

**Sol.** Let $4^0 = 1 =$ △, $4^1 = 4 =$ □, $4^2 = 16 =$ ○,

1 = △
2 = △△
3 = △△△
4 = □
5 = □△
6 = □△△
7 = □△△△
8 = □□
9 = □□△
10 = □□△△
11 = □□△△△
12 = □□□
13 = □□□△
14 = □□□△△
15 = □□□△△△
16 = ○

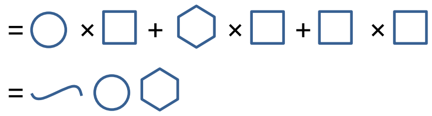
**3.** Give a simple rule to multiply a given number by 5 in the base-5 system that we created.

**Sol.** Rule: The Product of a landmark number with another landmark number gives a landmark number

□ × □ = ⬡; ⬡ × □ = ○; ○ × □ = ∼ etc.

Ex: ○⬡□ × □

= (○ + ⬡ + □) × □



## FIGURE IT OUT-08

**1.** Represent the following numbers in the Mesopotamian system:

(i) 63

(ii) 132

(iii) 200

(iv) 60

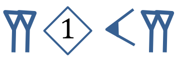
(v) 3605

**Sol.** (i) $63 = 60 + 3 = 1 \times 60 + 3$

= 

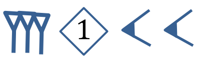
(ii) $132 = 1 \times 60 + 1 \times 60 + 10 + 2$

$= 2 \times 60 + 10 + 2$

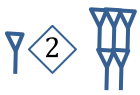
= 

(iii) $200 = 1 \times 60 + 1 \times 60 + 1 \times 60 + 20$

$= 3 \times 60 + 20$

= 

(iv) $60 = 1 \times 60$ = 

(v) $3605 = 1 \times 3600 + 5$ = 

## FIGURE IT OUT-09

**1.** Why do you think the Chinese alternated between the Zong and Heng symbols? If only the Zong symbols were to be used, how would 41 be represented? Could this numeral be interpreted in any other way if there is no significant space between two successive positions?

**Sol.** The most appropriate reason seems to be that they wanted to avoid misinterpretation while reading the number, as all numerals are represented by rods. Using only Zong symbols, 41 will be written as ||||.

This can easily be misinterpreted as 5 if no significant space is left between successive positions.

**2.** Form a base-2 place value system using 'ukasar' and 'urapon' as the digits. Compare this system with that of the Gumulgal's.

**Sol.** Let $2^0 = 1 = A$, $2^1 = 2 = B$, $2^2 = 4 = C$, $2^4 = 16 = D$,...

| Base 2-system | Gumulgal system |
|---|---|
| 1 : A | ur |
| 2 : B | uk |
| 3 : BA | uk - ur |
| 4 : C | uk - uk |
| 5 : CA | uk - uk - ur |
| 6 : CB | uk - uk - uk |
| 7 : CBA | uk - uk - uk - ur |
| 8 : D | uk - uk - uk - uk |

Both have the same base 2, but the base 2 system has many landmark numbers, whereas the Gumulgal system has only two landmark numbers.

**3.** Where in your daily lives, and in which professions, do the Hindu numerals, and 0, play an important role? How might our lives have been different if our number system and 0 hadn't been invented or conceived of?

**Sol.** They are useful wherever we have to read unit numbers or do any calculation.

In case '0' was not there, all the above would have become much tedious and cumbersome.

Also, there would have been no computers.

**4.** The ancient Indians likely used base 10 for the Hindu number system because humans have 10 fingers, and so we can use our fingers to count. But what if we had only 8 fingers? How would we be writing numbers then? What would the Hindu numerals look like if we were using base 8 instead? Base 5? Try writing the base-10 Hindu numeral 25 as base-8 and base-5 Hindu numerals, respectively. Can you write it in base-2?

**Sol.** For base 8, the numerals would have been: 0, 1, 2, 3, 4, 5, 6, 7

For base 5, the numerals would have been: 0, 1, 2, 3, 4

$$\begin{array}{r|l} 8 & 25 \\ \hline & 3 - 1 \\ \hline & \end{array}$$

$25_{10} = 31_8$

25 can be written as 31 in base 8.

$$\begin{array}{r|l} 5 & 25 \\ \hline & 5 - 0 \\ \hline & \end{array}$$

$25_{10} = 50_5$

25 can be written as 50 in base 5.

$$\begin{array}{r|l} 2 & 25 \\ \hline 2 & 12 - 1 \\ \hline 2 & 6 - 0 \\ \hline 2 & 3 - 0 \\ \hline & 1 - 1 \end{array}$$

$25_{10} = 11001_2$

25 can be written as 1101 in base 2.