

NCERT QUESTIONS WITH SOLUTIONS

FIGURE IT OUT

3.2 SUPERCELLS

1. Colour or mark the supercells in the table below.

6828	670	9435	3780	3708	8000	5583	52
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Sol.

6828	670	9435	3780	3708	8000	5583	52
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2. Fill the table below with only 4-digit numbers such that the supercells are exactly the coloured cells.

5346			1258				9635	
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Sol.

5346	6525	1000	1258	1056	1241	6325	9635	9754
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3. Fill the table below such that we get as many supercells as possible. Use numbers between 100 and 1000 without repetitions.

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Sol.

958	425	748	656	878	125	475	450	788
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4. Out of the 9 numbers, how many supercells are there in the table above?

Sol. Out of 9 numbers, there are 5 supercells in the above table.

5. Find out how many supercells are possible for different numbers of cells.

Do you notice any pattern? What is the method to fill a given table to get the maximum number of supercells? Explore and share your strategy.

Sol. If there are n odd cells then maximum number of supercells = $n+1/2$

If there are n even cells then maximum number of supercells = $n/2$

Yes, there is a pattern. Alternate cells can be supercells.

Method to fill a given table to get the maximum number of supercells.

Make first cell as supercell. After that each alternate cell is to be made supercell.

6. Can you fill a supercell table without repeating numbers such that there are no supercells? Why or why not?

Sol. No, it is not possible to fill a supercell table without repeating numbers such that there are no supercells. As there are two cases:

Case I: If we fill the cells in descending order then the first cell can be supercell.

Case II: If we fill the cells in ascending order then the last cell will be supercell. If we don't follow any order, then there will definitely at least one supercell.

7. Will the cell having the largest number in a table always be a supercell? Can the cell having the smallest number in a table be a supercell? Why or why not?

Sol. Yes, the cell having the largest number in a table always be a supercell (considering that the numbers in all the cells are different) because if it is a corner cell, then the number adjacent to it (i.e., either second cell or second last cell) will be smaller than it. If it is in between then both its adjacent numbers would be smaller than it.

No, the smallest number cannot be a supercell. A supercell has to be bigger than all its neighbours. Since it's the smallest, it can't be bigger than any neighbouring numbers, so it can't be a supercell.

8. Fill a table such that the cell having the second largest number is not a supercell.

Sol.

900	850	600	400	550	300	800	570
-----	-----	-----	-----	-----	-----	-----	-----

9. Fill a table such that the cell having the second largest number is not a supercell but the second smallest number is a supercell. Is it possible?

Sol.

700	651	400	450	350	200	100	150
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10. Make other variations of this puzzle and challenge your classmates.

Sol. We can also make other variations of given puzzle as:

A cell will be coloured if the number in it is smaller than its adjacent cells.

100	475	655	325	680	750	675	115	200	300
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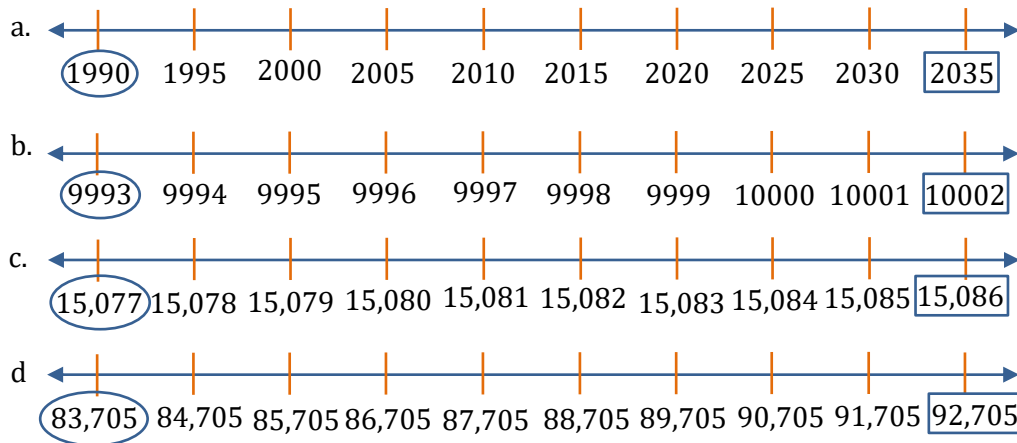
3.3 PATTERNS OF NUMBERS ON THE NUMBER LINE

1. Identify the numbers marked on the number lines below, and label the remaining positions.



Put a circle around the smallest number and a box around the largest number in each of the sequences above.

Sol.



3.4 PLAYING WITH DIGITS

1. Digit sum 14

- Write other numbers whose digits add up to 14.
- What is the smallest number whose digit sum is 14?
- What is the largest 5-digit number whose digit sum is 14?
- How big a number can you form having the digit sum 14? Can you make an even bigger number?

Sol. a. Some numbers whose digits add up to 14 are:

59, 77, 86, 95, 149, 158, 167, 185, 194, 239, 248, 257, 266, 275, 284, 293, 329,

b. The smallest number whose digit sum is 14 = 59.

c. The largest 5 digit number containing 0 whose digit sum is 14 = 95,000.

The largest 5 digit number not containing 0 whose digit sum is 14 = 92,111.

d. A very big number having the digit sum 14 can be made. E.g. 950000000000000.

Yes, we can make even bigger number. E.g. 950000000000000000000000000000.

2. Find out the digit sums of all the numbers from 40 to 70. Share your observations with the class.

Sol. Digit Sums from 40 to 70:

Number	Digit sum	Number	Digit Sum
40	4	55	10
41	5	56	11
42	6	57	12
43	7	58	13
44	8	59	14
45	9	60	6
46	10	61	7
47	11	62	8
48	12	63	9
49	13	64	10
50	5	65	11
51	6	66	12
52	7	67	13
53	8	68	14
54	9	69	15
		70	7

3. Calculate the digit sums of 3-digit numbers whose digits are consecutive (for example, 345). Do you see a pattern? Will this pattern continue?

Sol.

Number	123	234	345	456	567	678	789
Sum of digits	6	9	12	15	18	21	24

Yes, we observe a Pattern.

i.e., $(\text{First digit} + 1) \times 3 = \text{Digit sum}$.

For number 123,

$(1 + 1) \times 3 = 6 = \text{Digit sum}$

3.7 CLOCK AND CALENDAR NUMBERS

1. Pratibha uses the digits '4', '7', '3' and '2', and makes the smallest and largest 4-digit numbers with them: 2347 and 7432. The difference between these two numbers is $7432 - 2347 = 5085$. The sum of these two numbers is 9779. Choose 4-digits to make:
- The difference between the largest and smallest numbers greater than 5085.
 - The difference between the largest and smallest numbers less than 5085.
 - The sum of the largest and smallest numbers greater than 9779.
 - The sum of the largest and smallest numbers less than 9779.

Sol. a. Let's use the digits 2, 3, 1, 8:

Largest number: 8321

Smallest number: 1238

Difference = $8321 - 1238 = 7083$

Since 7083 is greater than 5085, this works.

- b. Let's use the digits 4, 5, 6, 8:

Largest number: 8654

Smallest number: 4568

Difference = $8654 - 4568 = 4086$

Since 4086 is less than 5085, this works.

- c. Let's use the digits 5, 9, 2, 3:

Largest number: 9532

Smallest number: 2359

Sum = $9532 + 2359 = 11891$

Since 11891 is greater than 9779, this works.

- d. Let's use the digits 7, 2, 5, 1:

Largest number = 7521

Smallest number = 1257

Sum = $7521 + 1257 = 8778$

Since 8778 is less than 9779, this works.

2. What is the sum of the smallest and largest 5-digit palindrome? What is their difference?

Sol. Smallest 5-Digit Palindrome: 10001.

Largest 5-Digit Palindrome: 99999.

Sum: $10001 + 99999 = 110000$.

Difference: $99999 - 10001 = 89998$.

3. The time now is 10:01. How many minutes until the clock shows the next palindromic time? What about the one after that?

Sol. The next palindromic time after 10:01 is 11:11.

The difference between 10:01 and 11:11 is 1 hour and 10 minutes, which equals to 70 minutes. The next palindromic time after 11:11 is 12:21.

The difference between 11:11 and 12:21 is also 1 hour and 10 minutes, or 70 minutes.

4. How many rounds does the number 5683 take to reach the Kaprekar constant?

Sol.

Round 1		Round 2		Round 3
8653	→	8550	→	9972
<u>- 3568</u>		<u>- 0558</u>		<u>- 2799</u>
<u>= 5085</u>		<u>= 7992</u>		<u>= 7173</u>
Round 4		Round 5		Round 6
7731	→	6543	→	8730
<u>- 1377</u>		<u>- 3456</u>		<u>- 0378</u>
<u>= 6354</u>		<u>= 3087</u>		<u>= 8352</u>
Round 7				
8532				
<u>- 2358</u>				
<u>= 6174</u>				

It takes 7 rounds for the number 5683 to reach the Kaprekar constant.

3.8 MENTAL MATH

1. Write an example for each of the below scenarios whenever possible.

5-digit + 5-digit to give a 5-digit sum more than 90,250	5-digit + 3-digit to give a 6-digit sum	4-digit + 4-digit to give a 6-digit sum	5-digit + 5-digit to give a 6-digit sum	5-digit + 5-digit to Give 18,500
5-digit – 5-digit to give a difference less than 56,503	5-digit – 3-digit to give a 4-digit difference	5-digit – 4-digit to give a 4-digit difference	5-digit – 5-digit to give a 3-digit difference	5-digit – 5-digit to to give 91,500

Could you find examples for all the cases? If not, think and discuss what could be the reason. Make other such questions and challenge your classmates.

Sol. Let's go through the scenarios presented and provided examples for each:

- 5-digit + 5-digit to give a 5-digit sum more than 90,250:
Example: $60,000 + 35,000 = 95,000$
- 5-digit + 3-digit to give a 6-digit sum:
Example: $99,250 + 750 = 1,00,000$
- 4-digit + 4-digit to give a 6-digit sum:
This seems impossible because two 4-digit numbers added together can't give a 6-digit number.
While adding two largest 4-digit number, the result is a 5-digit number ($9999 + 9999 = 19998$) so we can say 6-digit number sum is not possible.
- 5-digit + 5-digit to give a 6-digit sum:
Example: $70,000 + 45,000 = 1,15,000$
- 5-digit + 5-digit to give 18,500:
While adding two smallest 5-digit number (i.e., 10000), the result is a 20000 i.e., greater than 18500,
So, we can say 18500 is not possible.
- 5-digit – 5-digit to give a difference less than 56,503:
Example: $90,000 - 40,000 = 50,000$
- 5-digit – 3-digit to give a 4-digit difference:
Example: $10,000 - 600 = 9,400$
- 5-digit – 4-digit to give a 4-digit difference:
Example: $15,000 - 8,000 = 7,000$
- 5-digit – 5-digit to give a 3-digit difference:
Example: $11700 - 11000 = 700$
- 5-digit – 5-digit to give 91,500:
Example: $99999 - 10000 = 89999$ i.e., not equal to 91500
While difference of greatest 5-digit number and smallest 5-digit number is 89999
so, we can say required difference 91500 is not possible

Conclusion

We found examples for all the cases, except for

- Adding two 4-digit numbers to get a 6-digit sum, which is impossible. the largest sum is still a 5-digit number.
- Adding two 5-digit numbers to give 18500 as sum.
- Subtracting 5-digit number from another 5-digit number to give 91500 as sum.

2. Always, Sometimes, Never?

Below are some statements. Think, explore and find out if each of the statement is 'Always true', 'Only sometimes true' or 'Never true'. Why do you think so? Write your reasoning; discuss this with the class.

- 5-digit number + 5-digit number gives a 5-digit number
- 4-digit number + 2-digit number gives a 4-digit number
- 4-digit number + 2-digit number gives a 6-digit number
- 5-digit number – 5-digit number gives a 5-digit number
- 5-digit number – 2-digit number gives a 3-digit number

Sol. Checking Whether Always, Sometimes, Or Never:

- 5-digit number + 5-digit number gives a 5-digit number
Sometimes true
If both numbers are small, like $20,000 + 10,000 = 30,000$,
The result is a 5-digit number.
But with large numbers, like $95,000 + 95,000 = 190,000$, it becomes a 6-digit number.
- 4-digit number + 2-digit number gives a 4-digit number
Sometimes true
a small 4-digit number, like $2,000 + 70 = 2070$, gives a 4-digit result. but if the 4-digit number is large, like $9,999 + 99 = 10,098$, the result is a 5-digit number.
- 4-digit number + 2-digit number gives a 6-digit number
Never true
Even the largest 4-digit number (9,999) and the largest 2-digit number (99) only give a 5-digit result (10,098), not 6 digits.
So, 6 digit number is not possible.
- 5-digit number – 5-digit number gives a 5-digit number
Sometimes true
If they're far apart, like $99999 - 10,000 = 89999$, the result is a 5-digit number.
but if they're close, like $20,000 - 11000 = 9000$, the result is a 4-digit number.
- 5-digit number – 2-digit number gives a 3-digit number
Never true
If we take the smallest 5-digit number: 10,000 and the largest 2-digit number: 99 Then the difference = $10,000 - 99 = 9,901$ (a 4-digit number).

3.11 SIMPLE ESTIMATION

We shall do some simple estimates. It is a fun exercise, and you may find it amusing to know the various numbers around us. Remember, we are not interested in the exact numbers for the following questions. Share your methods of estimation with the class.

1. Steps you would take to walk:

- From the place you are sitting to the classroom door
- Across the school ground from start to end
- From your classroom door to the school gate
- From your school to your home

Sol. Do it yourself.

2. Number of times you blink your eyes or number of breaths you take:

- In a minute
- In an hour
- In a day

Sol. Do it yourself.

3. Name some objects around you that are:

- a few thousand in number
- more than ten thousand in number

Sol. Do it yourself.

3.12 GAMES AND WINNING STRATEGIES

1. There is only one supercell (number greater than all its neighbours) in this grid. If you exchange two digits of one of the numbers, there will be 4 supercells. Figure out which digits to swap.

16,200	39,344	29,765
23,609	62,871	45,306
19,381	50,319	38,408

Sol. Here, 62,871 is the only one supercell.

If we swap first and last digit of central number 62,871, we get desired result.

16,200	39,344	29,765
23,609	12,876	45,306
19,381	50,319	38,408

Now, 4 supercells are $\Rightarrow$ (39,344), (50,319), (45,306), (23,609).

2. How many rounds does your year of birth take to reach the Kaprekar constant?

Sol. My year of birth is 1995.

Round 1

Greatest number : 9951

Smallest number : 1599

Subtract: $9951 - 1599 = 8352$

Round 2

Greatest number : 8532

Smallest number : 2358

Subtract: $8532 - 2358 = 6174$

After 2 rounds, I reach the Kaprekar constant 6174.

3. We are the group of 5-digit numbers between 35,000 and 75,000 such that all of our digits are odd. Who is the largest number in our group? Who is the smallest number in our group? Who among us is the closest to 50,000?

Sol. Possible odd digits

Digits: 1, 3, 5, 7, 9

Largest Number: 73,999

Smallest Number: 35,111

Closest To 50,000: 51,111

4. Estimate the number of holidays you get in a year including weekends, festivals and vacation. Then try to get an exact number and see how close your estimate is.

Sol. Holidays in a Year

Weekends: 104 Days

Summer Vacation: 55 Days

Winter Vacation: 10 Days

Festival Holidays: 10 Days

Public Holidays: 5 Days

Total Estimated Holidays = $104 + 55 + 10 + 10 + 5 = 184$ Days.

5. Estimate the number of litres a mug, a bucket and an overhead tank can hold.

Sol. As per my house

Mug: 0.10 – 0.25 Litres

Bucket: 30 Litres

Overhead Tank: 3000 Litres

6. Write one 5-digit number and two 3-digit numbers such that their sum is 18,670.

Sol. 5-digit number: 17,355

3-digit number – 1: 784

3-digit number – 2: 531

Sum: $17,355 + 784 + 531 = 18,670$

7. Choose a number between 210 and 390. Create a number pattern similar to those shown in from NCERT will sum up to this number.

Sol. Simple Addition Pattern

We can create a simple addition pattern using a series of repeated numbers:

$30 + 30 + 30 + 30 + 60 + 60 + 60 = 300$

Pattern: 30 Repeated 4 Times, 60 Repeated 3 Times,

$30 + 30 + 30 + 30 = 120$

$60 + 60 + 60 = 180$

$120 + 180 = 300$

8. Recall the sequence of powers of 2 how student will relate. Why is the collatz conjecture correct for all the starting numbers in this sequence?

Sol. The square of power of 2 is:

1, 2, 4, 8, 16, 32, 64

Let's take the number 64 as per collatz conjecture.

64 is even, divide by 2 = 32

32 is even, divide by 2 = 16

16 is even, divide by 2 = 8

8 is even, divide by 2 = 4

4 is even, divide by 2 = 2 .

2 is even, divide by 2 = 1

Hence collatz conjecture is correct in all numbers in the power of 2 sequence. As it is power of 2, and in collatz conjecture even number is divided by 2 in each step.

9. Check if the Collatz conjecture holds for the starting number 100.

Sol. 100 is even, divide by 2 = 50.

50 is even, divide by 2 = 25

25 is odd, so multiply by 3 and add 1 $\rightarrow$ 76 and so on...

The sequence formed with starting number 100 is as follows: 100, 50, 25, 76, 38, 19, 58, 29, 88, 44, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1.

Yes, the Collatz conjecture holds for the starting number 100.

10. Starting with 0, players alternate by adding numbers between 1 and 3. The first person to reach 22 wins. What is the winning strategy now?

Sol. Do it yourself.
