

NCERT QUESTIONS WITH SOLUTIONS

FIGURE IT OUT

10.1 BELA'S BUILDING OF FUN

1. You start from Floor + 2 and press - 3 in the lift. Where will you reach? Write an expression for this movement.

Sol. Starting from Floor +2 and pressing -3 means moving down 3 floors.

Expression: $(2) + (-3) = -1$

You will reach Floor -1.

2. Evaluate these expressions (you may think of them as Starting Floor + Movement by referring to the Building of Fun).

a. $(+1) + (+4) = \underline{\hspace{2cm}}$

b. $(+4) + (+1) = \underline{\hspace{2cm}}$

c. $(+4) + (-3) = \underline{\hspace{2cm}}$

d. $(-1) + (+2) = \underline{\hspace{2cm}}$

e. $(-1) + (+1) = \underline{\hspace{2cm}}$

f. $0 + (+2) = \underline{\hspace{2cm}}$

g. $0 + (-2) = \underline{\hspace{2cm}}$

Sol. a. $(+1) + (+4) = +5$

b. $(+4) + (+1) = +5$

c. $(+4) + (-3) = +1$

d. $(-1) + (+2) = +1$

e. $(-1) + (+1) = 0$

f. $0 + (+2) = +2$

g. $0 + (-2) = -2$

3. Starting from different floors, find the movements required to reach Floor - 5. For example, if I start at Floor + 2, I must press - 7 to reach Floor - 5. The expression is $(+2) + (-7) = -5$. Find more such starting positions and the movements needed to reach Floor - 5 and write the expressions.

Sol. (a) If I start at floor +1, I must press (-6) to reach floor -5.

The expression is $(1) + (-6) = -5$.

(b) If I start at floor +3, I must press (-8) to reach the floor (-5).

The expression is $(+3) + (-8) = -5$.

10.1 BELA'S BUILDING OF FUN

Evaluate these expressions by thinking of them as the resulting movement of combining button presses:

a. $(+1) + (+4) = \underline{\hspace{2cm}}$

b. $(+4) + (+1) = \underline{\hspace{2cm}}$

c. $(+4) + (-3) + (-2) = \underline{\hspace{2cm}}$

d. $(-1) + (+2) + (-3) = \underline{\hspace{2cm}}$

Sol. a. Target floor = $(+1) + (+4) = +5$

b. Target floor = $(+4) + (+1) = +5$

c. Target floor = $(+4) + (-3) + (-2)$
 $= 4 + (-5) = -1$

d. Target floor = $(-1) + (+2) + (-3)$
 $= (-4) + (2) = -2$

10.1 BELA'S BUILDING OF FUN

1. Compare the following numbers using the Building or Fun and fill in the boxes with $<$ or $>$.

- a. $-2 \square +5$ b. $-5 \square +4$ c. $-5 \square -3$
 d. $+6 \square -6$ e. $0 \square -4$ f. $0 \square +4$

Notice that all negative number floors are below Floor 0. So, all negative numbers are less than 0. All the positive number floors are above 0. So, all positive numbers are greater than 0.

- Sol.** a. Here, floor -2 is lower than floor $+5$ then $-2 < +5$.
 b. Here, floor -5 is lower than floor $+4$ then $-5 < +4$.
 c. Here, floor -5 is lower than floor -3 then $-5 < -3$.
 d. Here, floor $+6$ is greater than floor -6 then $+6 > -6$.
 e. Here, floor 0 is greater than floor -4 then $0 > -4$.
 f. Here, floor 0 is lower than floor $+4$ then $0 < +4$.

2. Imagine the Building of Fun with more floors. Compare the numbers and fill in the boxes with $<$ or $>$.

- a. $-10 \square -12$ b. $+17 \square -10$ c. $0 \square -20$
 d. $+9 \square -9$ e. $-25 \square -7$ f. $+15 \square -17$

- Sol.** a. Here, floor -10 is greater than floor -12 then $-10 > -12$
 b. Here, floor $+17$ is greater than floor -10 then $+17 > -10$
 c. Here, floor 0 is greater than floor -20 then $0 > -20$
 d. Here, floor $+9$ is greater than floor -9 then $+9 > -9$
 e. Here, floor -25 is lower than floor -7 then $-25 < -7$
 f. Here, floor $+15$ is greater than floor -17 then $+15 > -17$

3. If Floor A = -12 , Floor D = -1 and Floor E = $+1$ in the building shown on the right as a line, find the numbers of Floors B, C, F, G, and H.

- Sol.** Let's mark the numbers on the line.

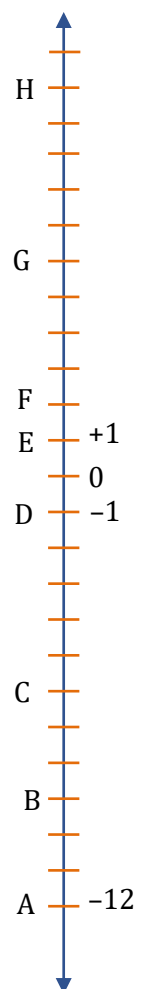
$0, -1, -2, -3, \dots, -12$ and $1, 2, 3, \dots, 12$

Now count each floor, and we get the number of floors

Here, B = -9 , C = -6 , F = $+2$, G = $+6$ and H = $+11$

4. Mark the following floors of the building shown on the right.

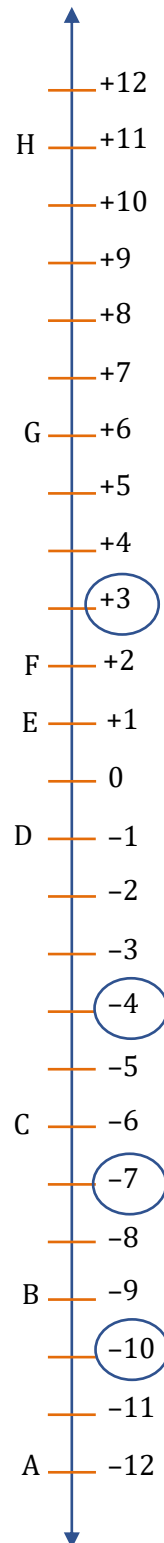
- a. -7
 b. -4
 c. $+3$
 d. -10



Sol. Here, one line is considered as one floor.

The floor above floor 0 is marked with positive numbers and the floor below 0 is numbered with negative numbers..

Now let's mark the following numbers $(-7, -4, +3, -10)$ in the building by circling the floor.



10.1 BELA'S BUILDING OF FUN

- ★ Complete these expressions. You may think of them as finding the movement needed to reach the Target Floor from the Starting Floor.

(a) $(+1) - (+4) =$ _____	(b) $(0) - (+2) =$ _____
(c) $(+4) - (+1) =$ _____	(d) $(0) - (-2) =$ _____
(e) $(+4) - (-3) =$ _____	(f) $(-4) - (-3) =$ _____
(g) $(-1) - (+2) =$ _____	(h) $(-2) - (-2) =$ _____
(i) $(-1) - (+1) =$ _____	(j) $(+3) - (-3) =$ _____

Sol.

(a) $(+1) - (+4) = \underline{-3}$	(b) $(0) - (+2) = \underline{-2}$
(c) $(+4) - (+1) = \underline{+3}$	(d) $(0) - (-2) = \underline{+2}$
(e) $(+4) - (-3) = \underline{+7}$	(f) $(-4) - (-3) = \underline{-1}$
(g) $(-1) - (+2) = \underline{-3}$	(h) $(-2) - (-2) = \underline{0}$
(i) $(-1) - (+1) = \underline{-2}$	(j) $(+3) - (-3) = \underline{+6}$

10.1 BELA'S BUILDING OF FUN

- ★ Complete these expressions.
- a. $(+40) + \underline{\hspace{2cm}} = +200$ b. $(+40) + \underline{\hspace{2cm}} = -200$
 c. $(-50) + \underline{\hspace{2cm}} = +200$ d. $(-50) + \underline{\hspace{2cm}} = -200$
 e. $(-200) - (-40) = \underline{\hspace{2cm}}$ f. $(+200) - (+40) = \underline{\hspace{2cm}}$
 g. $(-200) - (+40) = \underline{\hspace{2cm}}$

Check your answers by thinking about the movement in the mineshaft.

Sol. a. Given $(+40) + \underline{\hspace{2cm}} = +200$

Let $(+40) + x = +200$

$\Rightarrow +x = 200 - 40 = 160$

$\therefore (+40) + \underline{(+160)} = +200$

b. Given $(+40) + \underline{\hspace{2cm}} = -200$

Let $(+40) + x = -200$

$\Rightarrow x = -200 - 40 = -240$

$\therefore (+40) + \underline{(-240)} = -200$

c. Given $(-50) + \underline{\hspace{2cm}} = +200$

Let $(-50) + x = +200$

$\Rightarrow x = +200 - (-50) = +250$

$\therefore (-50) + \underline{(+250)} = +200$

d. Given $(-50) + \underline{\hspace{2cm}} = -200$

Let $(-50) + x = -200$

$\Rightarrow x = -200 - (-50) = -150$

$\therefore (-50) + \underline{(-150)} = -200$

e. Given $(-200) - (-40) = \underline{\hspace{2cm}}$

Let $(-200) - (-40) = x$

$\Rightarrow (-200) - (-40) = -160 = x$

$\therefore (-200) - (-40) = \underline{-160}$

f. Given $(+200) - (+40) = \underline{\hspace{2cm}}$

Let $(+200) - (+40) = x$

$\Rightarrow +160 = x$

$\therefore (+200) - (+40) = \underline{+160}$

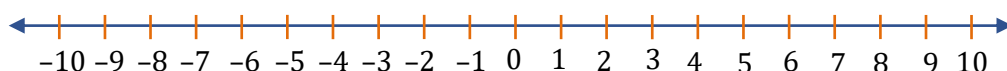
g. Given $(-200) - (+40) = \underline{\hspace{2cm}}$

Let $(-200) - (+40) = x$

$\Rightarrow (-200) + (-40) = -240 = x$

$\therefore (-200) - (+40) = \underline{-240}$

10.1 BELA'S BUILDING OF FUN

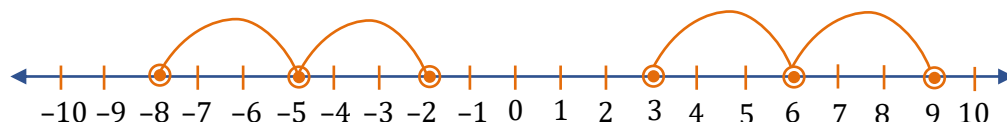


1. Mark 3 positive numbers and 3 negative numbers on the number line above.

Sol. On the Number Line:

Positive numbers: We can mark any three positive numbers, e.g., 3, 6 and 9.

Negative numbers: We can mark any three negative numbers, e.g., -2, -5 and -8.



2. Write down the above 3 marked negative numbers in the following boxes:

Sol. -2, -5 and -8 are three negative numbers.

3. Is $2 > -3$? Why? Is $-2 < 3$? Why?

Sol. Is $2 > -3$

Yes, 2 is a positive number and -3 is a negative number. We know that positive numbers are always greater than negative numbers.

Hence, 2 is greater than -3.

Is $-2 < 3$? Why?

$\therefore$ Yes, -2 is less than 3 because -2 is a negative number and 3 is a positive number.

Hence $-2 < 3$

4. What are

a. $-5 + 0$

b. $7 + (-7)$

c. $-10 + 20$

d. $10 - 20$

e. $7 - (-7)$

f. $-8 - (-10)?$

Sol. a. $-5 + 0$

Adding 0 to any number does not change the value of the number.

$$-5 + 0 = -5$$

b. $7 + (-7)$

Adding a number to its negative counterpart results in 0.

$$7 + (-7) = 0$$

c. $-10 + 20$

To add numbers with different signs, subtract the smaller absolute value from the larger absolute value and take the sign from the larger absolute value.

$$-10 + 20 = 10$$

d. $10 - 20$

Subtracting a larger number from a smaller number gives a negative result.

$$10 - 20 = -10$$

e. $7 - (-7)$

Subtracting a negative number is the same as adding the positive counterpart of the number.

$$7 - (-7) = 7 + 7 = 14$$

f. $-8 - (-10)$

Subtracting a negative number is the same as adding the positive counterpart of the number.

$$-8 - (-10) = -8 + 10 = 2$$

10.2 THE TOKEN MODEL

1. Complete the additions using tokens.

a. $(+6) + (+4)$

b. $(-3) + (-2)$

c. $(+5) + (-7)$

d. $(-2) + (+6)$

Sol. a. To represent $(+6)$, we use 6 positive tokens



and to represent $(+4)$, we use 4 positive tokens



Hence, by combining them, we get



$$\Rightarrow (+6) + (+4) = (+10)$$

$\therefore$ Counting all the positive tokens, we get $(+10)$.

b. To represent (-3) , we use 3 negative tokens



To represent (-2) , we use 2 negative tokens



Combining them, we get



$$\Rightarrow (-3) + (-2) = -5$$

$\therefore$ Counting all the negative tokens, we get (-5) .

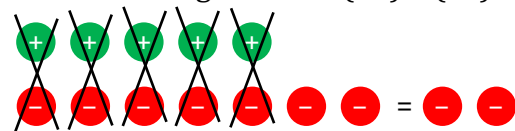
- c. To represent $(+5)$, we use 5 positive tokens



To represent (-7) , we use 7 negative tokens



$$\therefore \text{Remaining tokens} = (+5) + (-7) = -2$$



- d. To represent (-2) , we use 2 negative tokens



To represent $(+6)$, we use 6 positive tokens



Hence,



$$(-2) + (+6) = +4$$

Counting the remaining tokens, we get

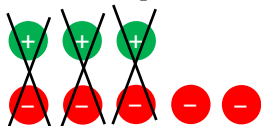


$(+4)$ tokens

2. Cancel the zero pairs in the following two sets of tokens. On what floor is the lift attendant in each case? What is the corresponding addition statement in each case?



- Sol.** a. From the picture we see that we can remove three pairs.

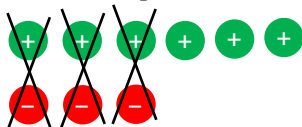


This cancels out to -2 .

$$\text{Remaining tokens} = \text{--} \text{--}$$

Since two negative tokens is remaining, the lift attendant is on the second floor below the ground floor. The corresponding addition statement is $(+3) + (-5) = (-2)$

- b. From the picture we see that we can remove three pairs.



This cancels out to $+3$.

$$\text{Remaining tokens} = \text{+ + +}$$

Since three positive tokens are remaining, the lift attendant is on the third floor above the ground floor.

The corresponding addition statement is $(+6) + (-3) = (+3)$

10.2 THE TOKEN MODEL

1. Evaluate the following differences using tokens. Check that you get the same result as with other methods you now know:

- a. $(+10) - (+7)$ b. $(-8) - (-4)$ c. $(-9) - (-4)$
 d. $(+9) - (+12)$ e. $(-5) - (-7)$ f. $(-2) - (-6)$

Sol. a. Here, from 10 positives take away 7 positives.



Hence, $(+10) - (+7) = +3$

b. Here, from -8 negatives take away -4 negatives



Hence, $(-8) - (-4) = -4$

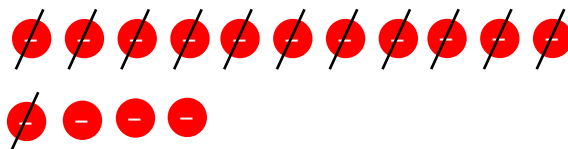
c. Here, from -9 negatives take away -4 negatives.



Hence, $(-9) - (-4) = -5$

d. There are not enough tokens to take out 12 positives from 9 positives.

So, we put an extra 3 zero pairs (1 pair = a positive and a negative). Now we can take out 12 positives.



Hence, $(+9) - (+12) = -3$

e. There are not enough tokens to take out -7 negatives from -5 negatives.

So, we put an extra two zero pairs (1 pair = a positive and a negative).

Now we can take out -7 negatives.



Hence, $(-5) - (-7) = +2$

f. There are not enough tokens to take out -6 negatives from -2 negatives.

So, we put an extra four zero pairs (1 pair = a positive and a negative).

Now we can take out -6 negatives.



Hence, $(-2) - (-6) = +4$

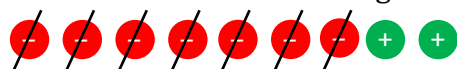
2. Complete the subtractions:

- a. $(-5) - (-7)$ b. $(+10) - (+13)$ c. $(-7) - (-9)$
 d. $(+3) - (+8)$ e. $(-2) - (-7)$ f. $(+3) - (+15)$

- Sol.** a. There are not enough tokens to take out -7 negatives from -5 negatives.

So, we put an extra two zero pairs (1 pair = a positive and a negative).

Now we can take out -7 negatives.

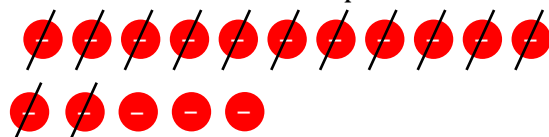


Hence, $(-5) - (-7) = +2$

- b. There are not enough tokens to take out 13 positives from 10 positives.

So, we put an extra 3 zero pairs (1 pair = a positive and a negative).

Now we can take out 13 positives.

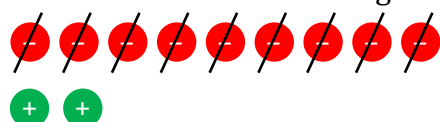


Hence, $(+10) - (+13) = -3$

- c. There are not enough tokens to take out -9 negatives from -7 negatives.

So, we put an extra two zero pairs (1 pair = a positive and a negative).

Now we can take out -9 negatives.

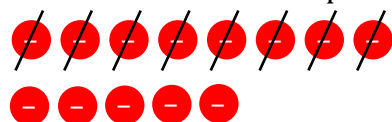


Hence, $(-7) - (-9) = +2$

- d. There are not enough tokens to take out 8 positives from 3 positives.

So, we put an extra 5 zero pairs (1 pair = a positive and a negative).

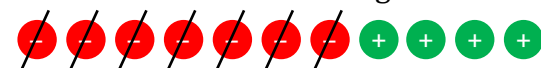
Now we can take out 8 positives.



Hence, $(+3) - (+8) = -5$

- e. There are not enough tokens to take out -7 negatives from -2 negatives. So, we put an extra two zero pairs (1 pair = a positive and a negative).

Now we can take out -7 negatives.

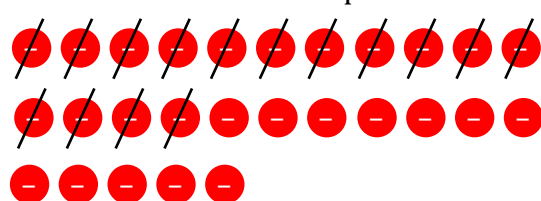


Hence, $(-2) - (-7) = +5$

- f. There are not enough tokens to take out 15 positives from 3 positives.

So, we put an extra 12 zero pairs (1 pair = a positive and a negative).

Now we can take out 15 positives.



Hence, $(+3) - (+15) = -12$

10.2 THE TOKEN MODEL

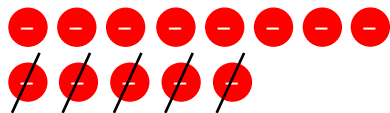
1. Try to subtract: $-3 - (+5)$.

How many zero pairs will you have to put in? What is the result?

Sol. We have to take out 5 positives from -3 negatives. But there are not enough positives.

So, we put down an extra 5 zero pairs (1 pair = a positive and a negative).

Now we can take out 5 positives.



Hence, $-3 - (+5) = -8$

2. Evaluate the following using tokens.

a. $(-3) - (+10)$ b. $(+8) - (-7)$ c. $(-5) - (+9)$

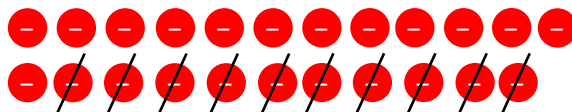
d. $(-9) - (+10)$ e. $(+6) - (-4)$ f. $(-2) - (+7)$

Sol. a. $(-3) - (+10)$

We have to take out 10 positives from -3 negatives. But there are not enough positives.

So, we put down extra 10 zero pairs (1 pair = a positive and a negative).

Now we can take out 10 positives.



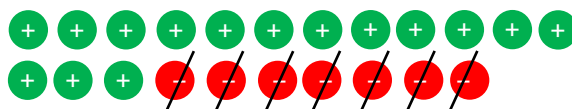
Hence, $(-3) - (+10) = -13$

b. $(+8) - (-7)$

We have to take out -7 negatives from 8 positives. But there are not enough negatives.

So, we put down an extra 7 zero pairs (1 pair = a positive and a negative).

Now we can take out -7 negatives.



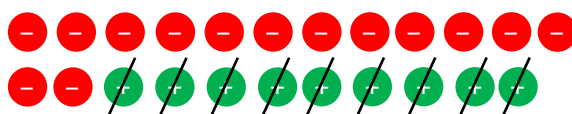
Hence, $(+8) - (-7) = 15$

c. $(-5) - (+9)$

We have to take out 9 positives from -5 negatives. But there are not enough positives.

So, we put down an extra 9 zero pairs (1 pair = a positive and a negative).

Now we can take out 9 positives.

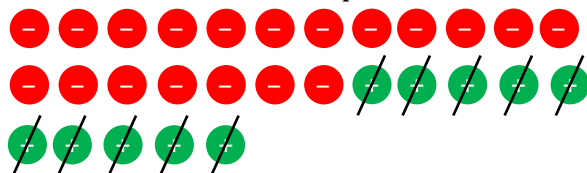


Hence, $-5 - (+9) = -14$

d. $(-9) - (+10)$

We have to take out 10 positives from -9 negatives. But there are not enough positives. So, we put down an extra 10 zero pairs (1 pair = a positive and a negative).

Now we can take out 10 positives.



Hence, $-9 - (+10) = -19$

e. $(+6) - (-4)$

We have to take out -4 negatives from 6 positives. But there are not enough negatives. So, we put down an extra 4 zero pairs (1 pair = a positive and a negative).

Now we can take out -4 negatives.

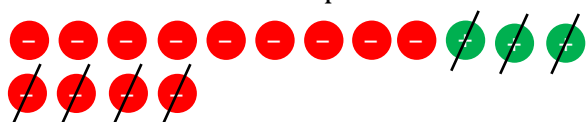


Hence, $(+6) - (-4) = 10$

f. $(-2) - (+7)$

We have to take out 7 positives from -2 negatives. But there are not enough positives. So, we put down an extra 7 zero pairs (1 pair = a positive and a negative).

Now we can take out 7 positives.



Hence, $(-2) - (+7) = -9$

10.3 INTEGERS IN OTHER PLACES

1. Suppose you start with ₹ 0 in your bank account, and then you have credits of ₹ 30, ₹ 40, and ₹ 50, and debits of ₹ 40, ₹ 50, and ₹ 60. What is your bank account balance now?

Sol. Here, Credits = ₹ 30 + ₹ 40 + ₹ 50 = ₹ 120

and Debits = ₹ 40 + ₹ 50 + ₹ 60 = ₹ 150

$\therefore$ Balance = Credits – Debits

= ₹ 120 – ₹ 150

= – ₹ 30

Therefore, your bank account balance is – ₹ 30.

2. Suppose you start with ₹ 0 in your bank account, and then you have debits of ₹ 1, 2, 4, 8, 16, 32, 64, and 128, and then a single credit of ₹ 256. What is your bank account balance now?

Sol. Here, Debits = ₹ 1 + ₹ 2 + ₹ 4 + ₹ 8 + ₹ 16 + ₹ 32 + ₹ 64 + ₹ 128 = ₹ 255

$\therefore$ Balance = Credits – Debits

= ₹ 256 – ₹ 255

= ₹ 1

Therefore, your bank account balance is ₹ 1.

3. Why is it generally better to try and maintain a positive balance in your bank account? What are circumstances under which it may be worthwhile to temporarily have a negative balance?

Sol. Having a positive balance in your bank account is generally better because:

- Avoids Overdraft Fees: Many banks charge fees if your account balance goes negative.
- Provide Financial Security: A positive balance ensures you have funds available for unexpected expenses or emergencies.
- Builds Good Credit History: Maintaining a positive balance can help to improve your credit score, making it easier to get loans or credit cards in the future.

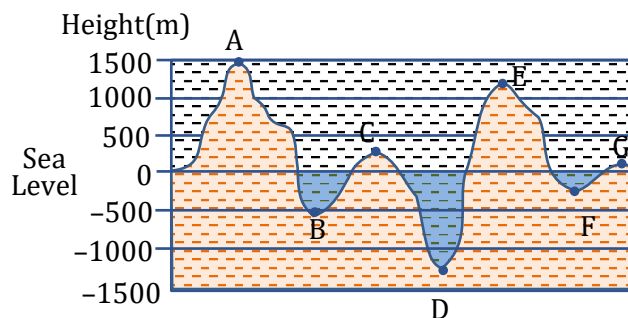
There might be a few specific situations where temporarily having a negative balance could be considered:

- Overdraft Protection: Some banks offer overdraft protection which can help avoid bounced checks or declared transactions.
- Planned Large Expenses: If you know you will have a large income soon and need to make essential purchases a temporary negative balance might be acceptable.

10.3 INTEGERS IN OTHER PLACES

1. Looking at the geographical cross section, fill in the respective heights:

A. B. C. D.
E. F. G.



Teacher's Note

Ask what a geographical cross section is by showing the figure in this page. It is like imagining a vertical slice taken out at some location on the Earth. This is what would be seen from a side view. Discuss the notion of 'sea level' for measuring heights and depths in geography.

- Sol.** You will need to estimate the height of each point A to G from the graph. Look at the vertical axis to determine the height value corresponding to each point.

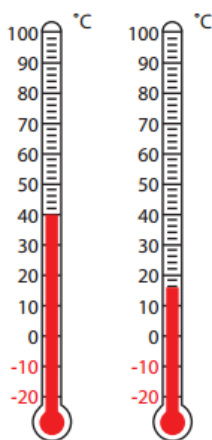
Heights:

- A = +1500 m
- B = -500 m
- C = +300 m
- D = -1200 m
- E = +1200 m
- F = -200 m
- G = +100 m

2. Which is the highest point in this geographical cross section? Which is the lowest point?
- Sol.** Identify the point on the graph that reaches the height point above sea level. Point A shows the highest point. Similarly find the point that is the lowest, considering points below sea level as well. Point D shows the lowest point.
3. Can you write the points A, B, ..., G in a sequence of decreasing order of heights? Can you write the points in a sequence of increasing order of heights?
- Sol.** Based on the heights we determined in question 1, arrange the points from highest to lowest (decreasing order) $A > E > C > G > F > B > D$ and then from lowest to highest (increasing order) $D < B < F < G < C < E < A$.
4. What is the highest point above sea level on Earth? What is its height?
- Sol.** The highest point on Earth is Mount Everest at a height of 8848 m above sea level.
5. What is the lowest point with respect to sea level on land or on the ocean floor? What is its height? (This height should be negative).
- Sol.** The lowest known point on the Earth is Marina Trench, in the Pacific Ocean, at a depth of 11034 m below sea level.

10.3 INTEGERS IN OTHER PLACES

1. Do you know that there are some places in India where temperatures can go below 0°C ? Find out the places in India where temperatures sometimes go below 0°C . What is common among these places? Why does it become colder there and not in other places?



Sol. Places:

- Ladakh: This region is well-known for its extremely cold winters, with temperatures often dropping below 0°C .
- Himachal Pradesh: Some high-altitude areas in Himachal Pradesh, especially the northern parts, can experience sub-zero temperatures.
- Jammu & Kashmir: Similar to Ladakh, parts of Jammu and Kashmir, particularly the mountainous regions, face freezing temperatures.
- Sikkim: Being a mountainous state, Sikkim also witness sub-zero temperatures in certain areas.
- Arunachal Pradesh: The higher reaches of Arunachal Pradesh can experience cold conditions with temperatures below 0°C .

Common Factor:

All these places are located in the Himalayan region, which is characterized by high altitudes.

Reason for Colder Temperatures:

- High Altitude: As altitude increases, the temperature decreases. This is primarily due to the thinner atmosphere at higher altitudes, which results in less heat retention.
- Distance from the Equator: These regions are farther from the equator receiving less direct sunlight, leading to colder temperatures.

2. Leh in Ladakh gets very cold during the winter. The following is a table of temperature readings taken during different times of the day and night in Leh on a day in November.

Match the temperature with the appropriate time of the day and night.

Temperature	Time
14°C	02:00 am
8°C	11:00 pm
-2°C	02:00 pm
-4°C	11:00 am

Sol.

Temperature	Time
14°C	02:00 am
8°C	11:00 pm
-2°C	02:00 pm
-4°C	11:00 am

10.4 EXPLORATIONS WITH INTEGERS

1. Do the calculations for the given grid and find the border sum

5	-3	-5
0		-5
-8	-2	7

Sol. Let's analyze the given grid and find the border sum.

Understanding the grid.

The given grid is a 3×3 arrangement of numbers.

The sum of numbers in each row and column should be the same.

Calculating the Border Sum:

Top row: $5 + (-3) + (-5) = -3$

Bottom row: $(-8) + (-2) + 7 = -3$

Left column: $5 + 0 + (-8) = -3$

Right column: $(-5) + (-5) + 7 = -3$

Therefore, the border sum of the given grid is -3

2. Complete the grids to make the required border sum:

-10		
		-5
9		7

6	8	
		-5
	-2	

7		
		-5

Border sum is +4 Border sum is -2 Border sum is -4

- Sol.** Here is the completed grid:

-10	12	2
5		-5
9	-12	7

The border sum is +4

Explanation:

We filled in the missing numbers to ensure the sum of each row and column is equal to the given border sum.

e.g. in the first grid, to get a border sum of +4, the missing number in the top row should be 12 and 2. (Since $-10 + 12 + 2 = 4$).

Attempt the remaining two grids by yourself.

3. For the last grid above, find more than one way of filling the numbers to get border sum - 4.

- Sol.** There are multiple ways to fill the last grid with a border sum of -4, here are two

examples:

7	-19	8
-2		-5
-9	12	-7

7	6	-17
8		-5
-19	5	18

4. Which other grids can be filled in multiple ways? What could be the reason?

- Sol.** Grid with a larger size (more rows and columns) is likely to have multiple solutions. This is because there are more degrees of freedom to distribute numbers while maintaining the border sum.

5. Make a border integer square puzzle and challenge your classmates.

- Sol.** Do it yourself

10.4 EXPLORATIONS WITH INTEGERS

1. Try afresh, choose different numbers this time. What sum did you get? Was it different from the first time? Try a few more times!

Sol. Let's circle the number 3.

Lets strike out the row and column with the number 3.

3	4	0	9
-2	-1	-5	4
1	2	-2	7
-7	-6	-10	-1

Again lets circle the number -1

Let's strike out the row and column with the number -1

3	4	0	9
-2	-1	-5	4
1	2	-2	7
-7	-6	-10	-1

Let's circle the number -5. Now as per the game, let's strike out the row and column with the number -5.

3	4	0	9
2	1	-5	4
1	2	-2	7
-7	-6	-10	-1

Let's circle the number 2. Now as per the game, let's strike out the row and column with the number 2.

3	4	0	9
2	1	-5	4
1	2	-2	7
-7	-6	-10	-1

Now let's add the circled numbers = $3 + (-1) + (-5) + 2 = -1$

Hence, we get the value (-1).

Now try yourself

2. Play the same game with the grids below. What answer did you get?

7	10	13	16	-11	-10	-9	-8
-2	1	4	7	-7	-6	-5	-4
-11	-8	-5	-2	-3	-2	-1	0
-20	-7	-14	-11	1	2	3	4

Sol. (a) Let's circle the number 1. Now as per the game, let's strike out the row and column with number 1.

7	10	13	16
-2	1	4	7
-11	-8	-5	-2
-20	-7	-14	-11

Again let's circle the number 13. Let's strike out the row and column with the number 13.

7	10	13	16
-2	1	4	7
-11	-8	-5	-2
-20	-7	-14	-11

Again let's circle the number -20. Let's strike out the row and column with the number -20.

7	10	13	16
-2	1	4	7
-11	-8	-5	-2
-20	-7	-14	-11

Again let's circle the number -2. Let's strike out the row and column with the number -2.

7	10	13	16
-2	1	4	7
-11	-8	-5	-2
-20	-7	-14	-11

Now let's add the circled numbers = $1 + 13 + (-20) + (-2) = -8$
which is the required answer.

(b) Let's strike out the row and column with the number -5.

Let's strike out the row and column with the number -5.

-11	-10	-9	-8
-7	-6	-5	-4
-3	-2	-1	0
1	2	3	4

Again let's circle the number 1.

Let's strike out the row and column with the number 1.

-11	-10	-9	-8
-7	-6	-5	-4
-3	-2	-1	0
1	2	3	4

Let's strike out the row and column with the number -10.

Again let's circle the balance number -10.

-11	-10	-9	-8
-7	-6	-5	-4
-3	-2	-1	0
1	2	3	4

Let's strike out the row and column with the number 0.

Again let's circle the balance number 0.

-11	-10	-9	-8
-7	-6	-5	-4
-3	-2	-1	0
1	2	3	4

Now let's add the circled numbers = $(-5) + 1 + (-10) + 0 = -14$ which is the required answer.

3. What could be so special about these grids? Is the magic in the numbers or the way they are arranged or both? Can you make more such grids?

Sol. Grids can be fascinating because of both the numbers and the way they are arranged. Here's why:

- Numbers: The numbers in a grid can follow specific patterns or sequences, such as magic squares where the sums of numbers in each row, column, and diagonal are the same.
- Arrangement: When you organize things in an orderly way, the result is called an arrangement. If you admire your friend's arrangement of his living room furniture, you might go home and make a similar arrangement

10.4 EXPLORATIONS WITH INTEGERS

1. Write all the integers between the given pairs, in increasing order.

- | | |
|-----------------|------------------|
| a. 0 and - 7 | b. - 4 and 4 |
| c. - 8 and - 15 | d. - 30 and - 23 |

- Sol.**
- The integers between 0 and -7, in increasing order, are: -6, -5, -4, -3, -2, -1
 - The integers between -4 and 4, in increasing order, are: -3, -2, -1, 0, 1, 2, 3
 - The integers between -8 and -15, in increasing order, are: -14, -13, -12, -11, -10, -9
 - The integers between -30 and -23, in increasing order, are: -29, -28, -27, -26, -25, -24

2. Give three numbers such that their sum is - 8.

Sol. Three numbers that add up to -8 are -10, -1, and 3.

When we add them together, we get $(-10) + (-1) + 3 = -8$

3. There are two dice whose faces have these numbers: - 1, 2, - 3, 4, - 5, 6. The smallest possible sum upon rolling these dice is $-10 = (-5) + (-5)$ and the largest possible sum is $12 = (6) + (6)$. Some numbers between (-10) and $(+12)$ are not possible to get by adding numbers on these two dice. Find those numbers.

Sol. Let's find the sums that are not possible when rolling these two dice.

The faces of the dice are: -1, 2, -3, 4, -5, and 6.

First, let's list all possible sums:

The sum of two negative numbers:

- $(-1) + (-1) = -2$
- $(-1) + (-3) = -4$
- $(-1) + (-5) = -6$
- $(-3) + (-3) = -6$
- $(-3) + (-5) = -8$
- $(-5) + (-5) = -10$

The sum of one negative and one positive number:

- $(-1) + 2 = 1$
- $(-1) + 4 = 3$
- $(-1) + 6 = 5$
- $(-3) + 2 = -1$
- $(-3) + 4 = 1$

- $(-3) + 6 = 3$
- $(-5) + 2 = -3$
- $(-5) + 4 = -1$
- $(-5) + 6 = 1$

The sum of two positive numbers:

- $2 + 2 = 4$
- $2 + 4 = 6$
- $2 + 6 = 8$
- $4 + 4 = 8$
- $4 + 6 = 10$
- $6 + 6 = 12$

Now, let's list all the possible sums in ascending order:

$-10, -8, -6, -4, -3, -2, -1, 1, 3, 4, 5, 6, 8, 10, 12$

The sum of numbers between -10 and 12 that are not possible to get are:

$-9, -7, -5, 0, 2, 7, 9, 11$

4. Solve these:

$8 - 13$	$(-8) - (13)$	$(-13) - (-8)$	$(-13) + (-8)$
$8 + (-13)$	$(-8) - (-13)$	$(13) - 8$	$13 - (-8)$

Sol. (a) $8 - 13 = -5$

(b) $-8 - (13) = -8 - 13 = -21$

(c) $-13 - (-8) = -13 + 8 = -5$

(d) $(-13) + (-8) = -13 - 8 = -21$

(e) $8 + (-13) = 8 - 13 = -5$

(f) $-8 - (-13) = -8 + 13 = 5$

(g) $(13) - 8 = 13 - 8 = 5$

(h) $13 - (-8) = 13 + 8 = 21$

5. Find the years below.

a. From the present year, which year was it 150 years ago? _____

b. From the present year, which year was it 2200 years ago? _____

Hint: Recall that there was no year 0.

c. What will be the year 320 years after 680 BCE? _____

Sol. a. 150 years ago from the present year (2024): $[2024 - 150 = 1874]$

So, 150 years ago, it was the year 1874.

b. 2200 years ago from the present year (2024):

Since there was no year 0, we need to account for this in our calculation:

$[2024 - 2200 = -176]$.

The year -176 corresponds to 177 BCE (Before the Common Era).

So, 2200 years ago, it was the year 177 BCE.

c. As BCE is before Christ, hence Let's write $680 \text{ BCE} = -680$

Hence, 320 years after 680 BCE $= -680 + 320 = -360 = 360 \text{ BCE}$.

6. Complete the following sequences:

- $(-40), (-34), (-28), (-22), \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}$
- $3, 4, 2, 5, 1, 6, 0, 7, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}$
- $\underline{\hspace{1cm}}, \underline{\hspace{1cm}}, 12, 6, 1, (-3), (-6), \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}$

Sol. a. Let's subtract a last number of the sequence with the preceding number.

$$\because (-22) - (-28) = -22 + 28 = 6$$

$$(-28) - (-34) = -28 + 34 = 6$$

$$-(-34) - (-40) = -34 + 40 = 6$$

Hence, this is a sequence where each term increases by 6.

$$\therefore \text{Next term} = -22 + 6 = -16$$

$$\therefore \text{Following term} = -16 + 6 = -10$$

$$\therefore \text{Final term} = -10 + 6 = -4$$

Hence, the sequence is $(-40), (-34), (-28), (-22), (-16), (-10), (-4)$

b. Let's subtract the last number of the sequence with the preceding number.

$$7 - 0 = 7$$

$$0 - 6 = -6$$

$$6 - 1 = 5$$

$$1 - 5 = -4$$

$$5 - 2 = 3$$

$$2 - 4 = -2$$

$$4 - 3 = 1$$

Hence in this sequence, numbers are decreasing by 1 with alternate positive and negative integers.

Hence the next number

$$7 + (-8) = -1$$

$$-1 + 9 = 8$$

$$8 - 10 = -2$$

$$-2 + 11 = 9$$

and so on.

Hence complete sequence is $3, 4, 2, 5, 1, 6, 0, 7, -1, 8, -2, 9, \dots$

Let us check

$$-1 - 7 = -8$$

$$8 + 1 = 9$$

$$-2 - 8 = -10$$

- c. Let's subtract the last number of the sequence from the preceding number

$$(-6) - (-3) = -6 + 3 = -3$$

$$(-3) - (1) = -3 - 1 = -4$$

$$1 - 6 = -5$$

$$6 - 12 = -6$$

Hence in this sequence, 1 negative integer is added to each number.

Let's take the first number of the sequence as x and the second number as y .

$$\text{2nd number} = 12 - y = -7$$

$$\text{Hence } y = 12 + 7 = 19$$

1st number, let it be x

$$\text{2nd number } 19 - x = -8$$

$$\Rightarrow x = 19 + 8 = 27$$

Now let's find the 8th number, let it be a .

$$\text{Hence, } a - (-6) = -2$$

$$\Rightarrow a = -2 - 6 = -8$$

Now let's find the 9th number, let it be b .

$$\text{Hence } b - (-8) = -1$$

$$\Rightarrow b = -1 - 8 = -9$$

Hence, the sequence is: 27, 19, 12, 6, 1, (-3), (-6), (-8), (-9), (-9)

7. Here are six integer cards: (+ 1), (+ 7), (+ 18), (- 5), (- 2), (- 9).

You can pick any of these and make an expression using addition(s) and subtraction(s).

Here is an expression: (+ 18) + (+ 1) - (+ 7) - (- 2) which gives a value (+ 14). Now, pick cards and make an expression such that its value is closer to (- 30).

- Sol.** Let's try to create an expression that gets as close to (-30) as possible using the given cards:

(+1, +7, +18, -5, -2, -9).

One possible expression is: (-9) + (-5) + (-2) - (-18) + (+1)

Let's calculate the value step by step:

$$1. \quad (-9) + (-5) = -14$$

$$2. \quad -14 + (-2) = -16$$

$$3. \quad -16 - (+18) = -34$$

$$4. \quad -34 + (+1) = -33$$

Hence, the value of this expression is (-33), which is quite close to (-30).

8. The sum of two positive integers is always positive but a (positive integer) - (positive integer) can be positive or negative. What about

a. (positive) - (negative)

b. (positive) + (negative)

c. (negative) + (negative)

d. (negative) - (negative)

e. (negative) - (positive)

f. (negative) + (positive)

Sol. a. (Positive) – (Negative):

Subtracting a negative number is the same as adding its positive counterpart. So, this will always be positive. For example, $5 - (-3) = 5 + 3 = 8$.

b. (Positive) + (Negative)

This depends on the magnitudes of the numbers. If the positive number is larger, the result is positive; if the negative number is larger, the result is negative.

For example,

$$7 + (-4) = 3 \text{ (positive)}$$

$$4 + (-7) = -3 \text{ (negative)}$$

c. (Negative) + (Negative)

Adding two negative numbers always results in a negative number.

For example, $-2 + (-3) = -5$.

d. (Negative) – (Negative)

This is like adding the positive counterpart of the second number to the first negative number.

If the first negative number is larger in magnitude, the result is negative. However, if the first negative number is smaller than the second negative number, then it is positive.

For example,

$$(-5) - (-2) = -3 \text{ (negative)}$$

$$(-2) - (-5) = 3 \text{ (positive)}$$

e. (Negative) – (Positive)

This will always be negative because you're subtracting a positive number from a negative number.

For example, $-4 - 2 = -6$.

f. (Negative) + (Positive)

Similar to (Positive) + (Negative), it depends on the magnitudes. If the positive number is larger, the result is positive; if the negative number is larger, the result is negative.

For example,

$$-3 + 5 = 2 \text{ (positive)}$$

$$-5 + 3 = -2 \text{ (negative)}$$

9. This string has a total of 100 tokens arranged in a particular pattern. What is the value of the string?



Sol. Let's analyze the sequence of the string:

3, -2, 3, -2, 3, -2



Let's take a set of 5 tokens as it is repeating, total is $3 - 2 = 1$

There are 100 tokens in the string.

Hence total sets = $100 \div 5 = 20$ sets

Total of 1 set = 1

Hence, the value of the string = $1 \times 20 = 20$

10.5 A PINCH OF HISTORY

1. Can you explain each of Brahmagupta's rules in terms of Bela's Building of Fun, or in terms of a number line?

Sol. Let's break down Brahmagupta's rules using the concept of Bela's Building of Fun and a number line. Brahmagupta's rules primarily deal with operations involving positive and negative numbers. Here's how we can understand them:

Brahmagupta's Rules

1. Addition of Positive Numbers:
 - Rule: Adding two positive numbers results in a positive number.
 - Bela's Building: If Bela starts on the 4th floor and moves up 2 floors, she ends up on the 6th floor.
 - Number Line: On a number line, moving from 4 to 6 by adding 2.
 - Example: $(4 + 2 = 6)$
2. Addition of Negative Numbers:
 - Rule: Adding two negative numbers results in a negative number.
 - Bela's Building: If Bela starts 5 floors below ground level (-5) and moves down 2 more floors, she ends up 7 floors below ground (-7).
 - Number Line: Moving from -5 to -7 by adding -2 .
 - Example: $((-5) + (-2) = -7)$
3. Addition of a Positive and a Negative Number:
 - Rule: Subtract the smaller absolute value from the larger absolute value and keep the sign of the larger absolute value.
 - Bela's Building: If Bela starts on the 6th floor and moves down 5 floors, she ends up 1 floor above ground (1).
 - Number Line: Moving from 6 to 1 by adding -5 .
 - Example: $(6 + (-5) = 1)$
4. Subtraction of a Positive Number from a Negative Number:
 - Rule: Subtracting a positive number from a negative number is like adding the two numbers and keeping the negative sign.
 - Bela's Building: If Bela is 3 floors below ground (-3) and moves down 3 more floors, she ends up 6 floors below ground (-6).
 - Number Line: Moving from -3 to -6 by subtracting 3.
 - Example: $(-3 - 3 = -6)$

5. Subtraction of a Negative Number from a Positive Number:

- Rule: Subtracting a negative number from a positive number is like adding two numbers.
- Bela's Building: If Bela starts on the 4th floor and moves up 2 floors, she ends up on the 6th floor.
- Number Line: Moving from 4 to 6 by subtracting -2 .
- Example: $(4 - (-2) = 6)$

6. Subtraction of a Negative Number from a Negative Number:

- Rule: Subtracting a negative number from another negative number is like adding the absolute values and keeping the negative sign.
- Bela's Building: If Bela is 7 floors below ground (-7) and moves up 2 floors, she ends up 5 floors below ground (-1).
- Number Line: Moving from -7 to -5 by subtracting -2 .
- Example: $-7 - (-2) = -5$

2. Give your own examples of each rule.

Sol. 1. Addition of Positive Numbers:

- Example: $(9 + 3 = 12)$
- Bela's Building: Starting on the 9th floor and moving up 3 floors to the 12th floor.

2. Addition of Negative Numbers:

- Example: $(-7 + (-3) = -10)$
- Bela's Building: Starting 7 floors below ground and moving down 3 more floors to 10 floors below ground.

3. Addition of a Positive and a Negative Number:

- Example: $(6 + (-3) = 3)$
- Bela's Building: Starting on the 8th floor and moving down 6 floors to the 2nd floor.

4. Subtraction of a Positive Number from a Negative Number:

- Example: $(-6 - 3 = -9)$
- Bela's Building: Starting 4 floors below ground and moving down 3 more floors to 7 floors below ground.

5. Subtraction of a Negative Number from a Positive Number:

- Example: $(5 - (-2) = 7)$
- Bela's Building: Starting on the 5th floor and moving up 2 floors to the 7th floor.

6. Subtraction of a Negative Number from a Negative Number:

- Example: $(-5 - (-3) = -2)$
- Bela's Building: Starting 5 floors below ground and moving up 3 floors to 2 floors below ground.