NCERT QUESTIONS WITH SOLUTIONS

FIGURE IT OUT

1.1 WHAT IS MATHEMATICS?

1. Can you think of other examples where mathematics helps us in our everyday lives?

Sol. Examples where mathematics helps in everyday life:

- Personal finance
- Sports and games
- Transport and travel

2. How has mathematics helped propel humanity forward? (You might think of examples involving: carrying out scientific experiments; running our economy and democracy; building bridges, houses or other complex structures; making tvs, mobile phones, computers, bicycles, trains, cars, planes, calendars, clocks, etc.)

Sol. **Scientific experiments:** Find mathematical relationships between the things they were observing.

Economy and democracy: To test theories, perform research, or understand trends and predictions for elections, counting votes and economic planning.

Building bridges, houses, or complex structures: Engineers use mathematics to ensure structural stability while building bridges or complex structures.

1.2 PATTERNS IN NUMBERS

1. Can you recognize the pattern in each of the sequences given below?

1, 1, 1, 1, 1, 1, 1, ...
1, 2, 3, 4, 5, 6, 7, ...
1, 3, 5, 7, 9, 11, 13, ...
2, 4, 6, 8, 10, 12, 14, ...
1, 3, 6, 10, 15, 21, 28, ...
1, 4, 9, 16, 25, 36, 49, ...
1, 8, 27, 64, 125, 216, ...
1, 2, 3, 5, 8, 13, 21, ...
1, 2, 4, 8, 16, 32, 64, ...
1, 3, 9, 27, 81, 243, 729, ...

Sol. Examples of number sequences

1, 1, 1, 1, 1, 1, 1, ...	(All 1's)
1, 2, 3, 4, 5, 6, 7,	(Counting numbers)
1, 3, 5, 7, 9, 11, 13, ...	(Odd numbers)
2, 4, 6, 8, 10, 12, 14, ...	(Even numbers)
1, 3, 6, 10, 15, 21, 28, ...	(Triangular numbers)
1, 4, 9, 16, 25, 36, 49, ...	(Squares)
1, 8, 27, 64, 125, 216, ...	(Cubes)
1, 2, 3, 5, 8, 13, 21,...	(Virahānka numbers)
1, 2, 4, 8, 16, 32, 64, ...	(Powers of 2)
1, 3, 9, 27, 81, 243, 729, ...	(Powers of 3)

2. Rewrite each sequence of table (given below) in your notebook, along with the next three numbers in each sequence! After each sequence, write in your own words what is the rule for forming the numbers in the sequence.

1, 1, 1, 1, 1, 1, 1, ...	(All 1's)
1, 2, 3, 4, 5, 6, 7,	(Counting numbers)
1, 3, 5, 7, 9, 11, 13, ...	(Odd numbers)
2, 4, 6, 8, 10, 12, 14, ...	(Even numbers)
1, 3, 6, 10, 15, 21, 28, ...	(Triangular numbers)
1, 4, 9, 16, 25, 36, 49, ...	(Squares)
1, 8, 27, 64, 125, 216, ...	(Cubes)
1, 2, 3, 5, 8, 13, 21,...	(Virahānka numbers)
1, 2, 4, 8, 16, 32, 64, ...	(Powers of 2)
1, 3, 9, 27, 81, 243, 729, ...	(Powers of 3)

Sol. Next three numbers in each sequence:

- All 1's: **1, 1, 1**

  In the sequence, Sequence of all 1 's.
- Counting numbers: 1, 2, 3, 4, 5, 6, 7, **8, 9, 10**

  In the sequence, A sequence of consecutive counting numbers starting from 1, adding 1 to the previous term to get the next term, as 1, 1 + 1 = 2, 2 + 1 = 3, 3 + 1 = 4 ……
- Odd numbers: 1,3, 5, 7, 9, 11, 13, **15, 17, 19**

  In the sequence, A sequence of consecutive odd numbers starting from 1, adding 2 to the previous term to get the next term, as 1, 1 + 2 = 3, 3 + 2 = 5, 5 + 2 = 7 ,7 + 2 = 9 ...
- Even numbers: 2, 4, 6, 8, 10, 12, 14, **16, 18, 20**

  In the sequence, A sequence of consecutive even numbers starting from 2, adding 2 to the previous term to get the next term, as 2, 2 + 2 = 4, 4 + 2 = 6, 6 + 2 = 8, 8 + 2 = 10…
- Triangular numbers: 1, 3, 6, 10, 15, 21, 28, **36, 45, 55**

  In the sequence, each term is the sum of first n consecutive counting numbers, as 1 = 1, 1 + 2 = 3, 1 + 2 + 3 = 6, 1 + 2 + 3 + 4 = 10, 1 + 2 + 3 + 4 + 5 = 15, 1 + 2 + 3 + 4 + 5 + 6 =21.
- Squares: 1, 4, 9, 16, 25, 36, 49, **64, 81, 100**

  In the sequence, each term is the product of counting number by itself starting from 1, as 1 × 1 = 1, 2 × 2 = 4, 3 × 3 = 9, 4 × 4 = 16, 5 × 5 = 25…

- Cubes: 1, 8, 27, 64, 125, 216, **343, 512, 729**

  In the sequence, each term is the product of counting number by itself thrice starting from 1, as 1 × 1 × 1 = 1 , 2 × 2 × 2 = 8, 3 × 3 × 3 = 27, 4 × 4 × 4 = 64, 5 × 5 × 5 = 125, 6 × 6 × 6 = 216....
- Virahānka numbers : 1, 2, 3, 5, 8, 13, 21, **34, 55, 89**

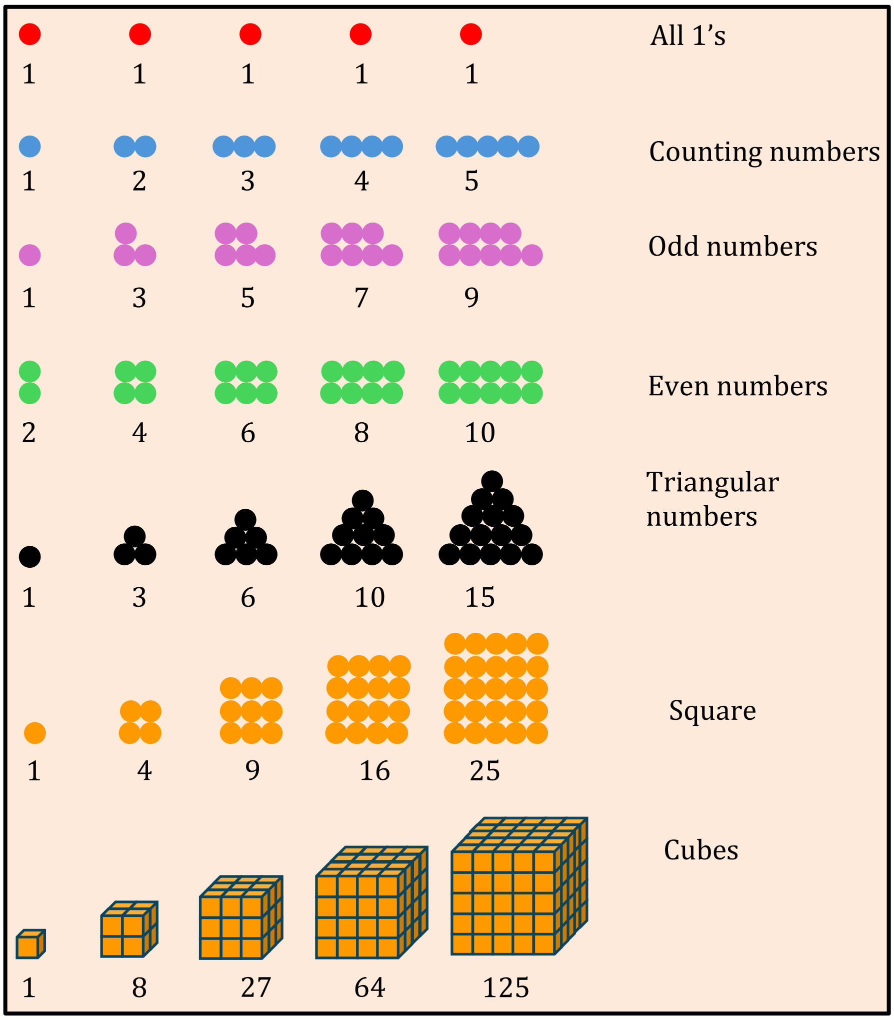
  In the sequence, each term (starting from third term) is the sum of previous two terms.
- Powers of 2: 1, 2, 4, 8, 16, 32, 64, **128, 256, 512**

  In the sequence, next term is the double of previous term, as 1, 1 × 2 = 2, 2 × 2 = 4, 4 × 2 = 8, 8 × 2 = 16, 16 × 2 = 32
- Powers of 3: 1, 3, 9, 27,81, 243,729, **2187, 6561, 19683**

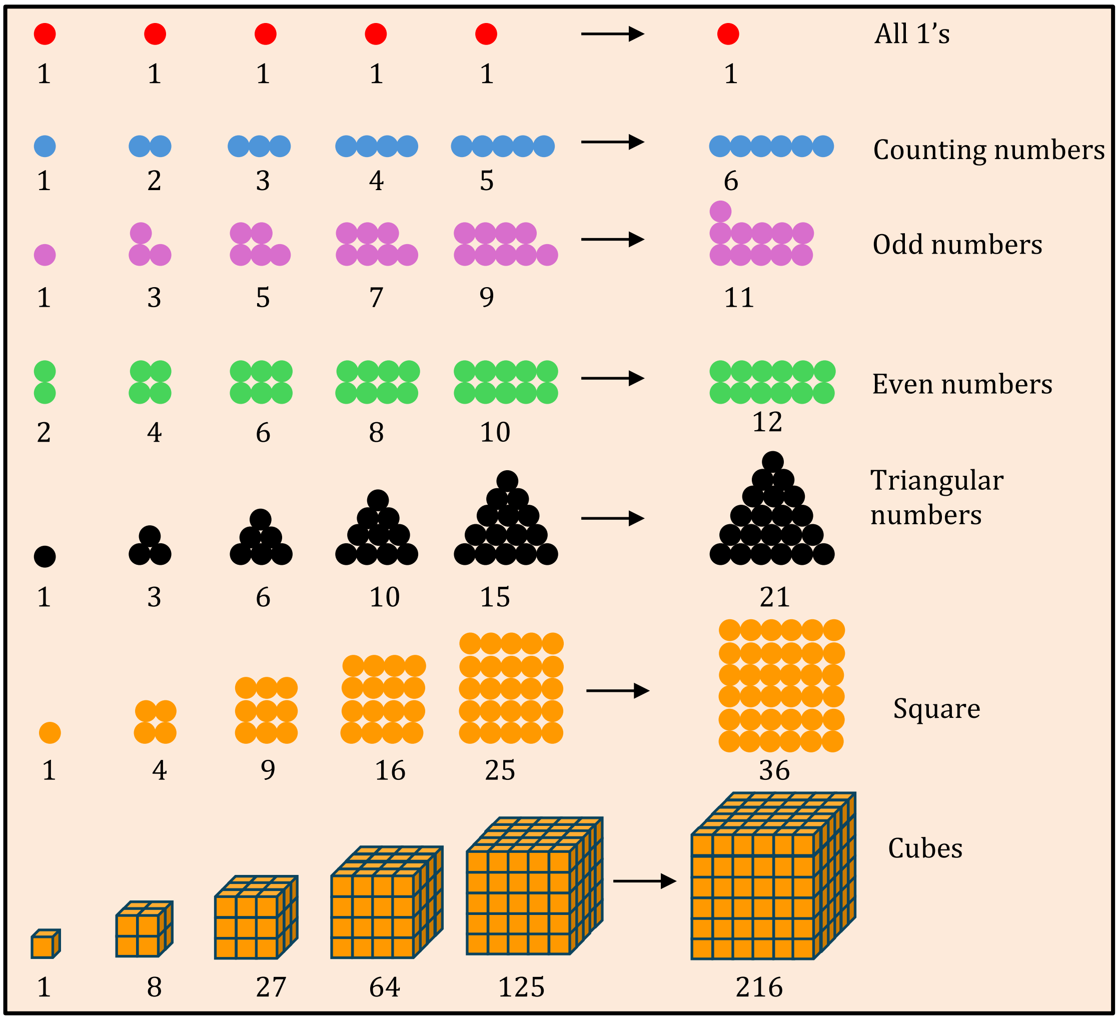
  In the sequence, next term is the thrice of previous term, as 1, 1 ×3 = 3, 3 ×3 = 9, 9 × 3 = 27, 27 × 3 = 81, 81 × 3 = 243, 243 × 3 = 729.

1.3 VISUALISING NUMBER SEQUENCES

1. Copy the pictorial representations of the number sequences in the table (given below) in your notebook, and draw the next picture for each sequence.



Sol.



2. Why are 1, 3, 6, 10, 15, called triangular numbers? Why are 1, 4, 9, 16, 25,... called square numbers or squares? Why are 1, 8, 27, 64, 125...... called cubes?

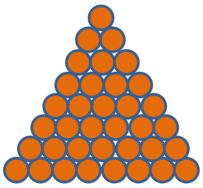
Sol. **Triangular numbers:** Each term is the sum of first n consecutive counting numbers, (1, 3, 6, 10, 15...). These numbers can form equilateral triangles when arranged in dots.

- 1, 4, 9, 16 are square numbers called so because they can form geometrical shape squares when arranged in dots.
- Cubes (1, 8, 27, 64...) are called cubes because they form the geometrical shape cube (shape having same length, breadth and height) when these number of unit cubes are arranged in a particular way.

3. You will have noticed that 36 is both a triangular number and a square number! That is, 36 dots can be arranged perfectly both in a triangle and in a square. Make pictures in your notebook illustrating this!

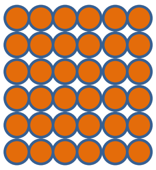
This shows that the same number can be represented differently, and play different roles, depending on the context. Try representing some other numbers pictorially in different ways!

Sol.



36 Dots

Triangular number

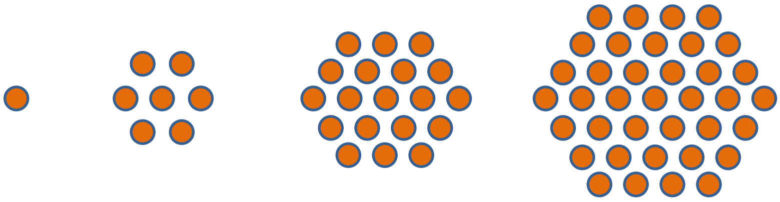


36 Dots

Square number

Some other same numbers that can be represented differently and play different roles that is 1225. (35 × 35). As it is both triangular as well as square number.

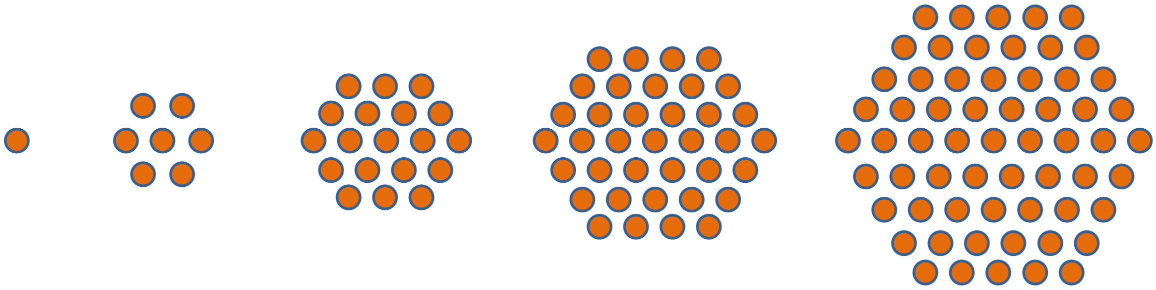
4. What would you call the following sequence of numbers?



1 7 19 37

That's right, they are called hexagonal numbers, Draw these in your notebook. What is the next number in the sequence?

Sol.



1 7 19 37 61

1^{st} number = 1

2^{nd} number = 1 + 6 = 7 (2^{nd} number = 1^{st} number + 6 × 1)

3^{rd} number = 7 + 12 = 19 (3^{rd} number = 2^{nd} number + 6 × 2)

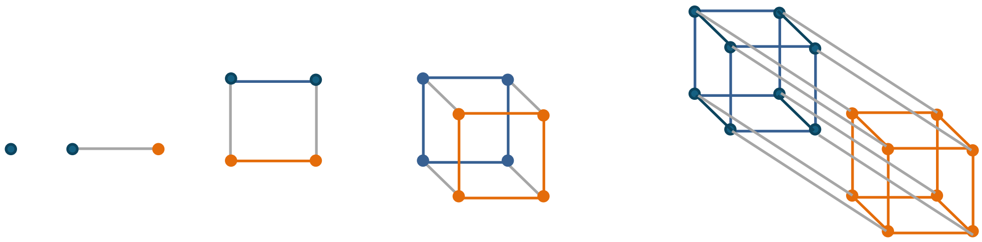
4^{th} number = 19 + 18 = 37 (4^{th} number = 3^{rd} number + 6 × 3)

5^{th} number = 37 + 24 = 61 (5^{th} number = 4^{th} number + 6 × 4)

Hence, the next number in the sequence is 61.

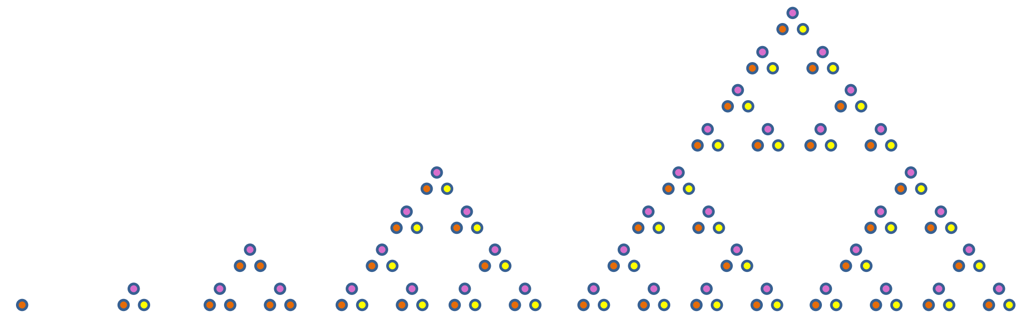
5. Can you think of pictorial ways to visualise the sequence of Powers of 2? Powers of 3?

Here is one possible way of thinking about Powers of 2:



1	2	4	8	16
2^0	2^1	2^2	2^3	2^4

Sol.



1 3 9 27 81

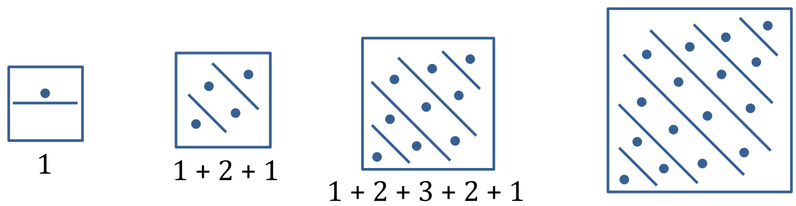
3^0 3^1 3^2 3^3 3^4

1.4 RELATIONS AMONG NUMBER SEQUENCES

1. Can you find a similar pictorial explanation for why adding counting numbers up and down, i.e., 1, 1 + 2 + 1, 1 + 2 + 3 + 2 + 1,..., gives square numbers?

Sol. When you add counting numbers up and down, like 1, 1 + 2 + 1, 1 + 2 + 3 + 2 + 1 etc., you are essentially forming symmetrical shapes that resemble squares.

For example:



1 1 + 2 + 1 1 + 2 + 3 + 2 + 1 1 + 2 + 3 + 4 +3 + 2 + 1

1 = 1 (1 square dot)

1 + 2 + 1 = 4 (forms a 2 × 2 square)

1 + 2 + 3 + 2 + 1 = 9 (forms a 3 × 3 square)

Each time, the numbers symmetrically increase and then decrease, giving a perfect square pattern.

2. By imagining a large version of your picture, or drawing it partially, as needed, can you see what will be the value of 1 + 2 + 3 +...+ 99 + 100 + 99 + ...+ 3 + 2 +1?

Sol.

1=1	(1^2)
1 + 2 + 1 = 4	(2^2)
1 + 2 + 3 + 2 + 1 = 9	(3^2)
1 + 2 + 3 + 4 + 3 + 2 + 1 = 16	(4^2)
1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1 = 25	(5^2)
1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3 + 2 + 1 = 36	(6^2)
1+2+3+.... +99 +100 + 99 +...+ 3 + 2 + 1=10000	(100^2)

3. Which sequence do you get when you start to add the All 1's sequence up? What sequence do you get when you add the All 1's sequence up and down?

Sol. When we add all 1's sequence up we get the counting numbers, as

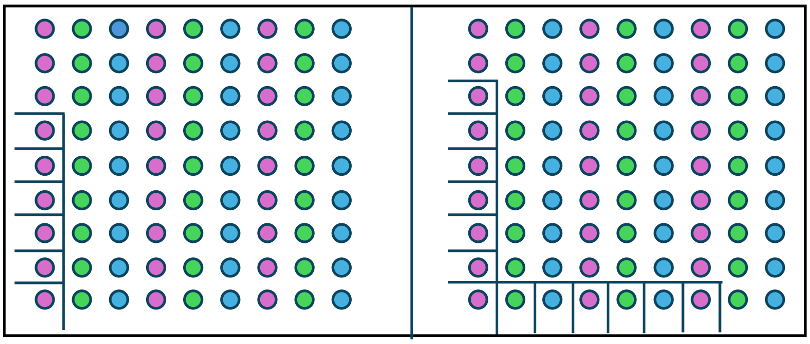
1 = 1,

1 + 1 = 2,

1 + 1 + 1 = 3,

1 + 1 + 1 +1 = 4,

When we add all 1's sequence up and down, we get counting numbers depend upon number of times 1 occurs.



4. Which sequence do you get when you start to add the Counting numbers up? Can you give a smaller pictorial explanation?

Sol. If you add the counting numbers (1, 2, 3, 4, ...), you get triangular numbers:

1 = 1

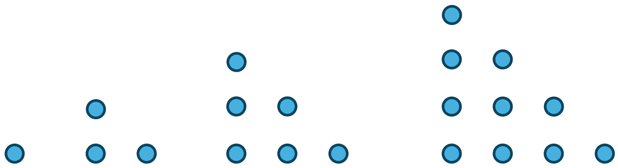
1 + 2 = 3

1 + 2 + 3 = 6

1 + 2 + 3 + 4 = 10

This forms the triangular number sequence.

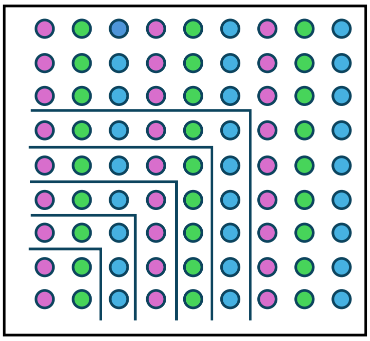
Pictorial representation:



1 3 6 10

5. What happens when you add up pairs of consecutive triangular numbers? That is, take 1 + 3, 3 + 6, 6 + 10, 10 + 15 ...? Which sequence do you get? Why? Can you explain it with a picture?

Sol.



When we add up pairs of consecutive triangular numbers i.e, 1+ 3, 3+6, 6+10, 10 + 15 we get square number sequence i.e.,

1 + 3 = 4 = 2 × 2

3 + 6 = 9 = 3 × 3

6 + 10 = 16 = 4 × 4 ....

6. What happens when you start to add up powers of 2 starting with 1, i.e., take 1, 1 + 2, 1 + 2 + 4, 1 + 2 + 4 + 8 ,...? Now add 1 to each of these numbers –what numbers do you get? Why does this happen?

Sol. When we start to add powers of 2:

1

$1 + 2 = 3$

$1 + 2 + 4 = 7$

$1 + 2 + 4 + 8 = 15$

When we add 1 to each of these numbers:

$1 + 1 = 2$

$3 + 1 = 4$

$7 + 1 = 8$

$15 + 1 = 16$

We get a number sequence of powers of 2 again: 2, 4, 8, 16, ...

7. What happens when you multiply the triangular numbers by 6 and add 1? Which sequence do you get? Can you explain it with a picture?

Sol. Multiplying the triangular numbers by 6 and adding 1 gives:

$1 \times 6 + 1 = 7$

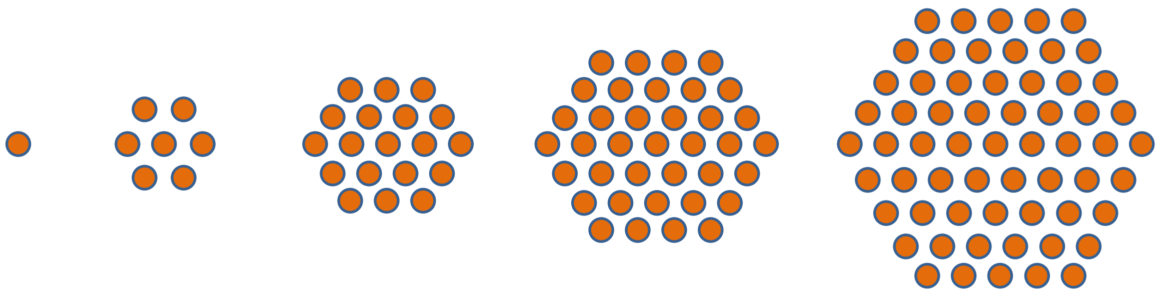
$3 \times 6 + 1 = 19$

$6 \times 6 + 1 = 37$

$10 \times 6 + 1 = 61$

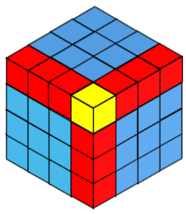
This forms the hexagonal number sequence: 7, 19, 37, 61 ...

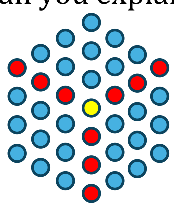
Pictorial representation:



1 7 19 37 61

8. What happens when you start to add up hexagonal numbers, i.e., take 1, 1 +7, 1 + 7 + 19, 1 + 7 + 19 + 37 ,...? Which sequence do you get? Can you explain it using a picture of a cube?





Sol. When we start to add up hexagonal numbers,

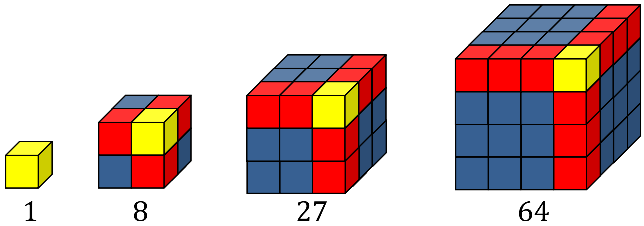
$1 = 1 \times 1 \times 1 = 1^3$

$1+7 = 8 = 2 \times 2 \times 2 = 2^3$

$1 + 7 + 19 = 27 = 3 \times 3 \times 3 = 3^3$

$1 + 7 + 19 + 37 = 64 = 4 \times 4 \times 4 = 4^3$

We get the cube of consecutive numbers. i.e., 1^3, 2^3, 3^3, 4^3, ...



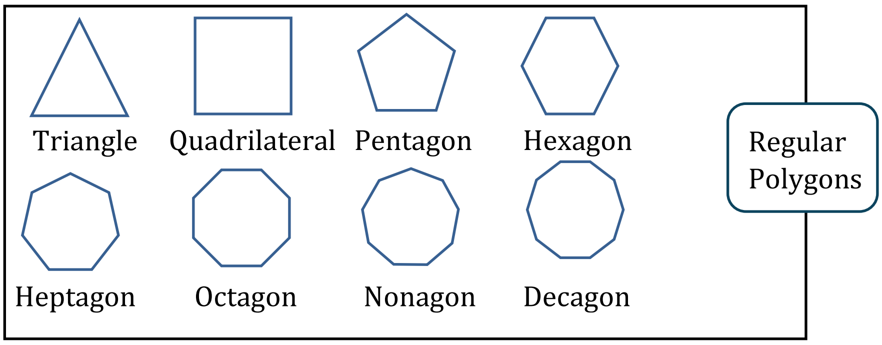
9. Find your patterns or relations in and among the sequences in Table 1. Can you explain why they happen with a picture or otherwise?

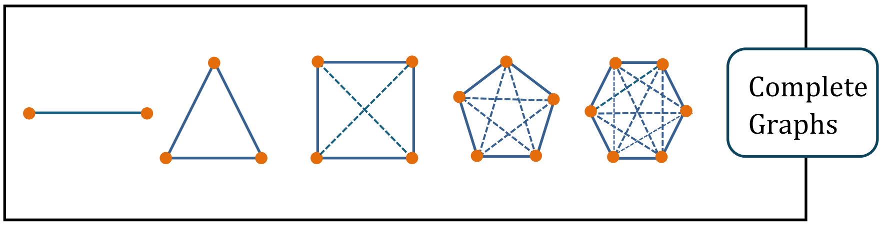
Sol. 3, 6, 9, 12, 15, 18.......... (consecutive multiples of 3).

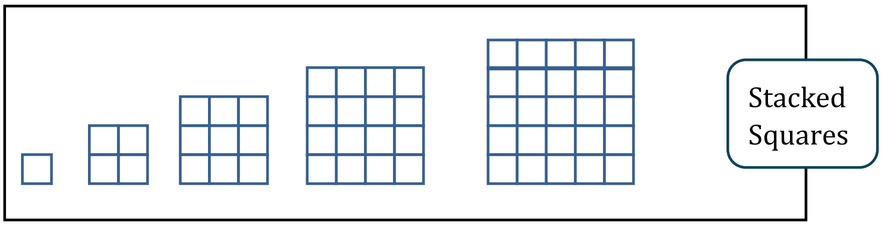
10, 15, 20, 25......... (first number is 10. Then increase of 5 in each term)

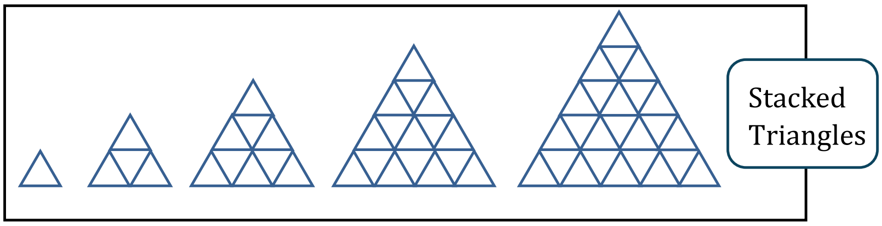
1.5 PATTERNS IN SHAPES

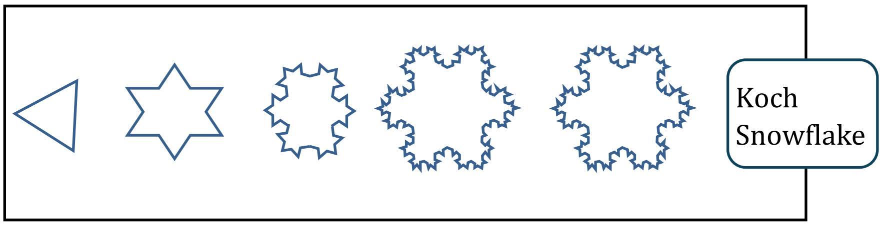
1. Can you recognize the pattern in each of the sequences in the table given below?











Sol. **Regular Polygons:** Triangle, quadrilateral, pentagon, hexagon, heptagon, etc.

Pattern: The number of sides increases by 1 each time, forming a polygon with an additional side.

Complete Graphs: K2, K3, K4, K5, etc.

Pattern: The number of vertices increases by 1, and the lines connecting every vertex form a complete graph. The number of edges increases accordingly.

Stacked Squares: Squares are stacked upon each other, with additional layers of smaller squares.

Pattern: More squares are added as layers, increasing the total number of smaller squares.

Stacked Triangles: Triangles stacked upon each other, increasing the number of small triangles in the structure.

Pattern: As new layers are added, the number of smaller triangles increases.

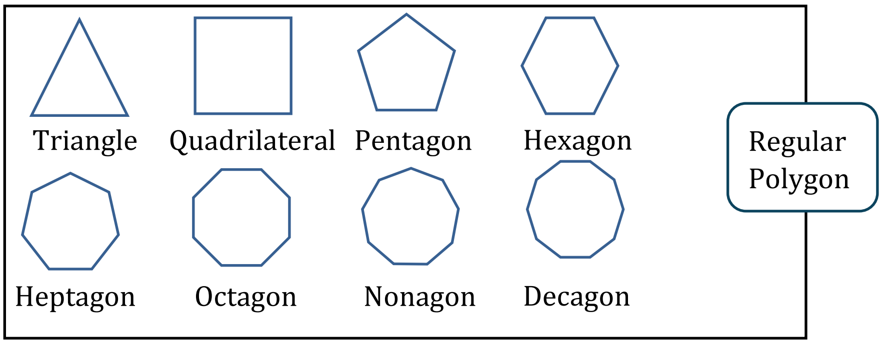
Koch Snowflake : As fractal pattern where each line segment is replaced by smaller "bumps" in the shape of an equilateral triangle.

Pattern: Each iteration adds more bumps along the edges, increasing the complexity and the number of line segments.

2. Try and redraw each sequence in the table given in question 1, in your notebook. Can you draw the next shape in each sequence? Why or why not? After each sequence, describe in your own words what is the rule or pattern for forming the shapes in the sequence.

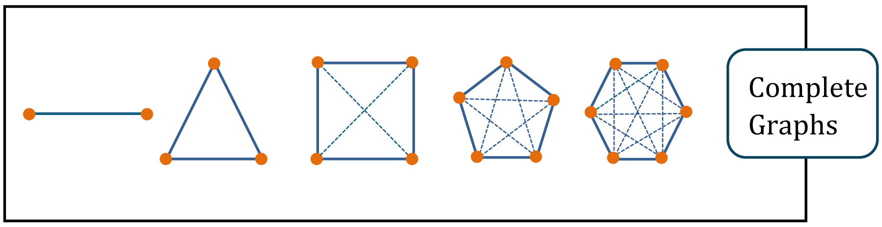
Sol. **Regular Polygon:** The next shape after the Decagon (10 sides) is the Hendecagon (11 sides).

Rule: Increase the number of sides by 1



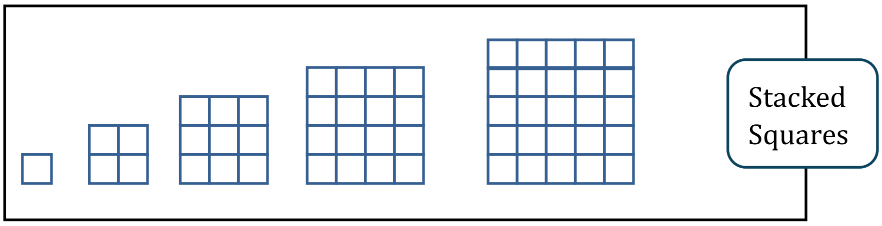
Complete graphs: After K6 (6 vertices), the next complete graph is K7

Rule: Add one more vertex and connect every vertex to all others,



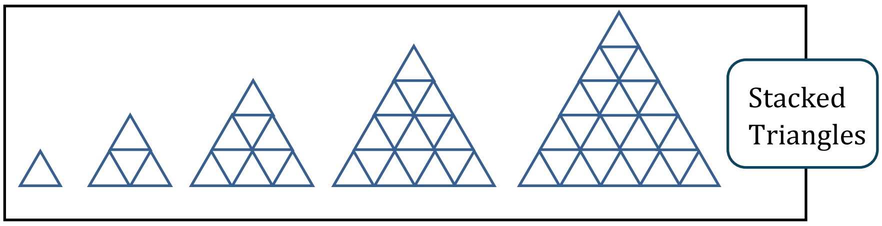
Stacked Squares: The next shape will have an additional layer of squares stacked below or around the existing structure

Rule: Add another layer with additional small squares, expanding the structure.



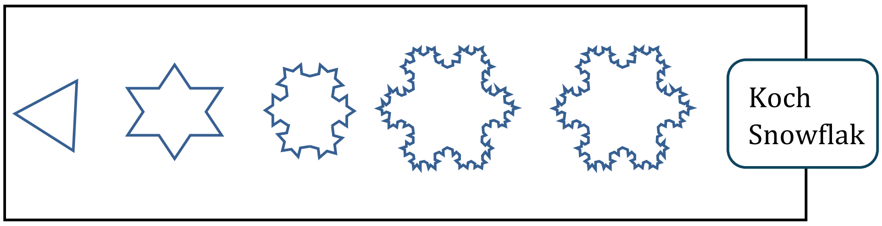
Stacked Triangles: The next shape will have one more layer of triangles at the base, increasing the total number of triangles.

Rule: Add another row of triangles to form a larger slacked structure.



Koch Snowflake: The next shape will have more intricate and smaller triangular "bumps" added to each side of the snowflake.

Rule: Each line segment is replaced with 4 smaller segments (one segment for the bump), increasing the total number of segments exponentially.



1.6 RELATION TO NUMBER SEQUENCES

1. Count the number of sides in each shape in the sequence of Regular Polygons. Which number sequence do you get? What about the number of corners in each shape in the sequence of Regular Polygons? Do you get the same number sequence? Can you explain why this happens?

Sol. Sides and Corners Sequence:

The number of sides and corners in each shape of the regular polygon sequence increases by 1.

Triangle: 3 sides, 3 corners

Quadrilateral: 4 sides, 4 corners

Pentagon: 5 sides, 5 corners

Hexagon: 6 sides, 6 corners

Heptagon: 7 sides, 7 corners

Number Sequence: 3, 4, 5, 6, 7...

The number of sides and corners is the same because each corner corresponds to a side in regular polygons.

2. Count the number of lines in each shape in the sequence of Complete Graphs. Which number sequence do you get? Can you explain why?

Sol. Number of Lines in Complete Graphs Sequence :

The number of lines in a complete graph (Kn) increases based on how many vertices are connected.

K2: 1 line

K3: 3 lines

K4: 6 lines

K5: 10 lines

3. How many little squares are there in each shape of the sequence of Stacked Squares? Which number sequence does this give? Can you explain why?

Sol. Number of Squares Sequence : As squares are stacked, the number of little squares increases:

1^{st} shape: 1 square

2^{nd} shape: 4 squares

3^{rd} shape: 9 squares

4^{th} shape: 16 squares

Number Sequence: 1, 4, 9, 16...

The number of squares follows the sequence of square numbers, as each shape forms a perfect square grid.

4. How many little triangles are there in each shape of the sequence of Stacked Triangles? Which number sequence does this give? Can you explain why? (Hint: In each shape in the sequence, how many triangles are there in each row?)

Sol. Number of little triangles sequence of Stacked Triangles = 1, 4, 9, 16, 25

5. To get from one shape to the next shape in the Koch Snowflake sequence, one replaces each line segment '-'by a 'speed bump'. As one does this more and more times, the changes become tinier and tinier with very very small line segments. How many total line segments are there in each shape of the Koch Snowflake? What is the corresponding number sequence? (The answer is 3, 12, 48, ..., i.e. 3 times Powers of 4).

Sol. Koch Snowflake Line Segments Sequence:

1^{st} shape: 3 line segments

2^{nd} shape: 12 line segments

3^{rd} shape: 48 line segments

Number Sequence: 3, 12, 48, ...

The number of line segments increases by multiplying the previous number by 4, forming a sequence of 3 times powers of 4.