NCERT QUESTIONS WITH SOLUTIONS

EXERCISE : 8.1

1. In ΔABC, right angled at B, AB = 24 cm, BC = 7 cm. Determine : (i) sin A, cos A (ii) sin C, cos C.

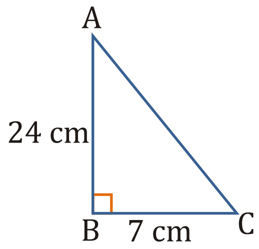
Sol. By Pythagoras Theorem,

$AC^2 = AB^2 + BC^2 = (24)^2 + (7)^2 = 625$

$\Rightarrow$ $AC = \sqrt{625} = 25$ cm.

(i) $\sin A = \dfrac{BC}{AC} \left\{ \text{i.e., } \dfrac{\text{side opposite to angle A}}{\text{Hyp.}} \right\}$

$= \dfrac{7}{25}$ ($\because$ BC = 7 cm and AC = 25 cm)



$\cos A = \dfrac{AB}{AC} \left\{ \text{i.e., } \dfrac{\text{side adjacent to angle A}}{\text{Hyp.}} \right\}$

$= \dfrac{24}{25}$ ($\because$ AB = 24 cm and AC = 25 cm)

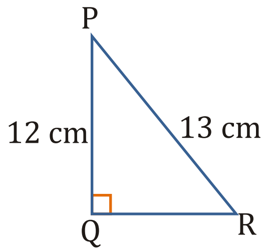
(ii) $\sin C = \dfrac{AB}{AC} \left\{ \text{i.e., } \dfrac{\text{side opposite to angle C}}{\text{Hyp.}} \right\}$

$= \dfrac{24}{25}$ $\cos C = \dfrac{BC}{AC}$

$\left\{ \text{i.e., } \dfrac{\text{side adjacent to angle C}}{\text{Hyp.}} \right\}$

$= \dfrac{7}{25}$

2. In fig, find tan P – cot R.



Sol. In figure, by the Pythagoras Theorem,

$QR^2 = PR^2 - PQ^2 = (13)^2 - (12)^2 = 25$

$\Rightarrow$ $QR = \sqrt{25} = 5$ cm

In ΔPQR right angled at Q, QR = 5 cm is side opposite to the angle P and PQ = 12 cm is side adjacent to the angle P.

Therefore, $\tan P = \dfrac{QR}{PQ} = \dfrac{5}{12}$.

Now, QR = 5 cm is side adjacent to the angle R and PQ = 12 cm is side opposite to the angle R.

Therefore, $\cot R = \dfrac{QR}{PQ} = \dfrac{5}{12}$

Hence, $\tan P - \cot R = \dfrac{5}{12} - \dfrac{5}{12} = 0$

3. If $\sin A = \dfrac{3}{4}$, calculate cos A and tan A.

Sol. In figure,

$\sin A = \dfrac{3}{4}$

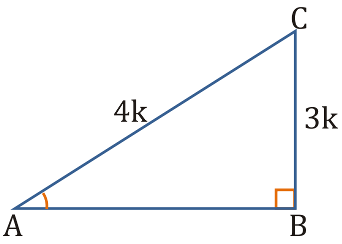
$\Rightarrow$ $\dfrac{BC}{AC} = \dfrac{3}{4}$

$\Rightarrow$ BC = 3k

and AC = 4k

where k is the constant of proportionality.

By Pythagoras Theorem,



$AB^2 = AC^2 - BC^2 = (4k)^2 - (3k)^2 = 7k^2$

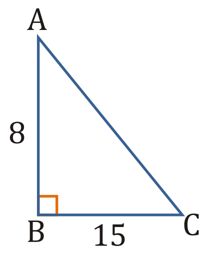
$\Rightarrow$ $AB = \sqrt{7}\,k$

So, $\cos A = \dfrac{AB}{AC} = \dfrac{\sqrt{7}k}{4k} = \dfrac{\sqrt{7}}{4}$

and $\tan A = \dfrac{BC}{AB} = \dfrac{3k}{\sqrt{7}k} = \dfrac{3}{\sqrt{7}}$

4. Given 15 cot A = 8, find sin A and sec A.

Sol.



$\cot A = \dfrac{8}{15}$

$\Rightarrow$ $\dfrac{AB}{BC} = \dfrac{8}{15}$

$\Rightarrow$ $AB = 8k$

and $BC = 15k$

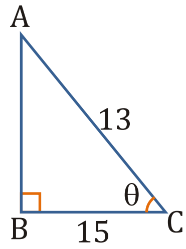
Now, $AC = \sqrt{(8k)^2 + (15k)^2} = 17k$

$\sin A = \dfrac{BC}{AC} = \dfrac{15k}{17k} = \dfrac{15}{17}$

$\sec A = \dfrac{AC}{AB} = \dfrac{17k}{8k} = \dfrac{17}{8}$

5. Given $\sec\theta = \dfrac{13}{12}$, calculate all other trigonometric ratios.

Sol.



$\sec\theta = \dfrac{13}{12}$

$\Rightarrow$ $\dfrac{AC}{BC} = \dfrac{13}{12}$

By Pythagoras Theorem,

$AC^2 = AB^2 + BC^2$

$(13k)^2 = AB^2 + (12k)^2$

$AB^2 = 169k^2 - 144k^2$

$AB = \sqrt{25k^2} = 5k$

$\sin\theta = \dfrac{AB}{AC} = \dfrac{5k}{13k} = \dfrac{5}{13}$

$\cos\theta = \dfrac{BC}{AC} = \dfrac{12k}{13k} = \dfrac{12}{13}$

$\tan\theta = \dfrac{AB}{BC} = \dfrac{5k}{12k} = \dfrac{5}{12}$

$\cot\theta = \dfrac{BC}{AB} = \dfrac{12k}{5k} = \dfrac{12}{5}$

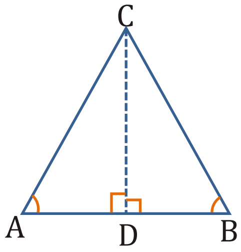
$\operatorname{cosec}\theta = \dfrac{AC}{AB} = \dfrac{13k}{5k} = \dfrac{13}{5}$

6. If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, then show that $\angle A = \angle B$.

Sol. In figure $\angle A$ and $\angle B$ are acute angles of ΔABC.

Draw $CD \perp AB$.

We are given that cos A = cos B



$\Rightarrow \quad \frac{AD}{AC} = \frac{BD}{BC} = k$ ..(1)

AD = kAC

BD = kBC

In ΔACD

$CD = \sqrt{AC^2 - AD^2}$

$CD = \sqrt{AC^2 - k^2AC^2}$

$CD = AC\sqrt{1-k^2}$ ... (2)

In ΔBCD

$CD = \sqrt{BC^2 - BD^2}$

$CD = \sqrt{BC^2 - k^2BC^2}$

$CD = BC\sqrt{1-k^2}$ ... (3)

equation (2) and (3)

$\frac{CD}{CD} = \frac{AC}{BC}$

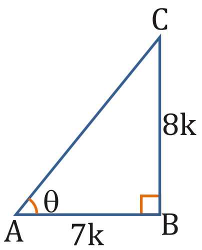
AC = BC

$\Rightarrow \quad \angle A = \angle B$ (Angles opposite to equal sides are equal)

7. If $\cot\theta = \frac{7}{8}$ evaluate :

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ (ii) $\cot^2\theta$

Sol. In figure,



$\cot\theta = \frac{7}{8}$

$\Rightarrow \quad \frac{AB}{BC} = \frac{7}{8}$

$\Rightarrow \quad AB = 7k$

and BC = 8k

Now, $AC^2 = AB^2 + BC^2 = (7k)^2 + (8k)^2$

$= 113\ k^2$

$\Rightarrow \quad AC = \sqrt{113}\ k$

Then $\sin\theta = \frac{BC}{AC} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$

and $\cos\theta = \frac{AB}{AC} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$.

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} =$

$$\frac{\left(1+\frac{8}{\sqrt{113}}\right)\left(1-\frac{8}{\sqrt{113}}\right)}{\left(1+\frac{7}{\sqrt{113}}\right)\left(1-\frac{7}{\sqrt{113}}\right)}$$

$$\frac{(\sqrt{113}+8)(\sqrt{113}-8)}{(\sqrt{113}+7)(\sqrt{113}-7)} = \frac{(\sqrt{113})^2-(8)^2}{(\sqrt{113})^2-(7)^2}$$

$\{\because (a+b)(a-b) = a^2 - b^2\}$

$= \frac{113-64}{113-49} = \frac{49}{64}$

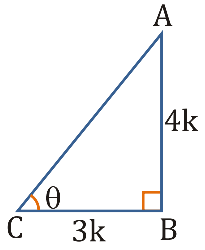
(ii) $\cot\theta = \frac{7}{8}$

$\Rightarrow \quad \cot^2\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$

8. If 3 cot A = 4, check whether

$\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Sol. In figure,



$3 \cot A = 4$

$\Rightarrow \cot A = \dfrac{4}{3}$

$\Rightarrow \dfrac{AB}{BC} = \dfrac{4}{3}$

$\Rightarrow$ AB = 4k and BC = 3k

Now, $AC = \sqrt{(4k)^2 + (3k)^2} = 5k$

Then $\sin A = \dfrac{BC}{AC} = \dfrac{3k}{5k} = \dfrac{3}{5}$,

$\cos A = \dfrac{AB}{AC} = \dfrac{4k}{5k} = \dfrac{4}{5}$

and $\tan A = \dfrac{BC}{AB} = \dfrac{3k}{4k} = \dfrac{3}{4}$

$$\text{LHS} = \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2}$$

$$= \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{16-9}{16+9} = \frac{7}{25}$$

$$\text{RHS} = \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

Therefore, LHS = RHS,

i.e., $\dfrac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

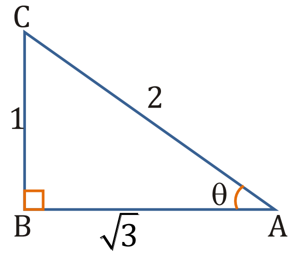
$\left(\because \text{Each side} = \dfrac{7}{25}\right)$

9. In triangle ABC right angled at B, if $\tan A = \dfrac{1}{\sqrt{3}}$, find the value of :

(i) sin A cos C + cos A sin C

(ii) cos A cos C – sin A sin C.

Sol.



$\tan A = \dfrac{1}{\sqrt{3}}$

$\dfrac{BC}{BA} = \dfrac{1}{\sqrt{3}}$

BC = k and $BA = \sqrt{3}k$

$AC^2 = BC^2 + BA^2$

$= k^2 + (\sqrt{3}k)^2 = k^2 + 3k^2 = 4k^2$

$AC = \sqrt{4k^2} = 2k$

(i) sin A cos C + cos A sin C

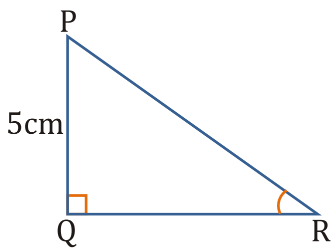
$= \dfrac{1}{2} \times \dfrac{1}{2} + \dfrac{\sqrt{3}}{2} \times \dfrac{\sqrt{3}}{2} = \dfrac{1}{4} + \dfrac{3}{4} = 1$

(ii) cos A . cos C – sin A . sin C

$= \dfrac{\sqrt{3}}{2} \times \dfrac{1}{2} - \dfrac{1}{2} \times \dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{3}}{4} - \dfrac{\sqrt{3}}{4} = 0$

10. In ΔPQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

Sol. In figure,



PQ = 5 cm

PR + QR = 25 cm

i.e., PR = 25 cm – QR

Now, $PR^2 = PQ^2 + QR^2$

$\Rightarrow$ $(25 - QR)^2 = (5)^2 + QR^2$

$\Rightarrow$ $625 - 50 \times QR + QR^2 = 25 + QR^2$

$\Rightarrow$ $50 \times QR = 600 \Rightarrow QR = 12$ cm

and PR = 25 cm – 12cm = 13 cm

We find $\sin P = \frac{QR}{PR} = \frac{12}{13}$, $\cos P = \frac{PQ}{PR} = \frac{5}{13}$

and $\tan P = \frac{QR}{PQ} = \frac{12}{5}$

11. State whether the following are true or false. Justify your answer.

(i) The value of tan A is always less than 1.

(ii) $\sec A = \frac{12}{5}$ for some value of angle A.

(iii) cos A is the abbreviation used for the cosecant of angle A.

(iv) cot A is the product of cot and A.

(v) $\sin\theta = \frac{4}{3}$ for some angle θ.

Sol. (i) False

We know that, $\tan 60° = \sqrt{3} > 1$.

(ii) True

We know that, value of sec A is always ≥ 1.

(iii) False

Because cos A is abbreviation used for cosine A.

(iv) False, because cot A is not the product of cot and A.

(v) False, because value of sin θ cannot be more than 1.

EXERCISE : 8.2

1. Evaluate :

(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

(iii) $\dfrac{\cos 45°}{\sec 30° + \text{cosec } 30°}$

(iv) $\dfrac{\sin 30° + \tan 45° - \text{cosec } 60°}{\sec 30° + \cos 60° + \cot 45°}$

(v) $\dfrac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

Sol. (i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{4} + \frac{1}{4} = 1$$

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

$$= 2 \times (1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 + \frac{3}{4} - \frac{3}{4} = 2$$

(iii) $\dfrac{\cos 45°}{\sec 30° + \text{cosec } 30°}$

$$= \frac{1/\sqrt{2}}{\frac{2}{\sqrt{3}} + 2} = \frac{1/\sqrt{2}}{2\left(\frac{1+\sqrt{3}}{\sqrt{3}}\right)} = \frac{1(\sqrt{3})}{2\sqrt{2}(1+\sqrt{3})}$$

$$= \frac{\sqrt{3}}{2(\sqrt{2})} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{\sqrt{3}(\sqrt{3}-1)}{2\sqrt{2} \times 2}$$

$$= \frac{(3-\sqrt{3})}{4\sqrt{2}} = \frac{(3-\sqrt{3})}{4\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}-\sqrt{6}}{8}$$

(iv) $\dfrac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

$$= \dfrac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1}$$

$$= \dfrac{\frac{\sqrt{3} + 2\sqrt{3} - 4}{2\sqrt{3}}}{\frac{4 + \sqrt{3} + 2\sqrt{3}}{2\sqrt{3}}} = \dfrac{(3\sqrt{3} - 4)}{(4 + 3\sqrt{3})}\dfrac{(4 - 3\sqrt{3})}{(4 - 3\sqrt{3})}$$

$$= \dfrac{12\sqrt{3} - 27 - 16 + 12\sqrt{3}}{16 - 9 \times 3}$$

$$= \dfrac{24\sqrt{3} - 43}{-11} = \dfrac{43 - 24\sqrt{3}}{11}$$

(v) $\dfrac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

$$= \dfrac{5(\cos 60^\circ)^2 + 4(\sec 30^\circ)^2 - (\tan 45^\circ)^2}{(\sin 30^\circ)^2 + (\cos 30^\circ)^2}$$

$$= \dfrac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \dfrac{\frac{5}{4} + 4 \times \frac{4}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \dfrac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{1}{4} + \frac{3}{4}} = \frac{5}{4} + \frac{16}{3} - 1$$

$$= \dfrac{15 + 64 - 12}{12} = \dfrac{67}{12}$$

2. Choose the correct option and justify your choice:

(i) $\dfrac{2\tan 30^\circ}{1 + \tan^2 30^\circ} =$

(A) $\sin 60^\circ$ (B) $\cos 60^\circ$

(C) $\tan 60^\circ$ (D) $\sin 30^\circ$

(ii) $\dfrac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} =$

(A) $\tan 90^\circ$ (B) 1

(C) $\sin 45^\circ$ (D) 0

(iii) $\sin 2A = 2 \sin A$ is true when $A =$

(A) 0° (B) 30°

(C) 45° (D) 60°

(iv) $\dfrac{2\tan 30^\circ}{1 - \tan^2 30^\circ} =$

(A) $\cos 60^\circ$ (B) $\sin 60^\circ$

(C) $\tan 60^\circ$ (D) $\sin 30^\circ$

Sol. (i) Option (A)

$$\dfrac{2\tan 30^\circ}{1 + \tan^2 30^\circ} = \dfrac{2\left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \dfrac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{\sqrt{3}}{2} = \sin 60^\circ$$

(ii) Option (D)

$$\dfrac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} = \dfrac{1 - 1}{1 + 1} = 0$$

(iii) Option (A)

By option 0°

$\sin 2A = \sin 0^\circ = 0$

$2\sin 0^\circ = 2 \times 0 = 0$

(iv) Option (C)

$$\dfrac{2\tan 30^\circ}{1 - \tan^2 30^\circ}$$

$$= \dfrac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \dfrac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \dfrac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$= \tan 60^\circ$

3. If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$; $0° < A + B \leq 90°$; $A > B$, find A and B.

Sol. $\tan(A + B) = \sqrt{3} \Rightarrow A + B = 60°$ ...(1)

$\tan(A - B) = \frac{1}{\sqrt{3}} \Rightarrow A - B = 30°$ ...(2)

Adding (1) and (2),

$2A = 90° \Rightarrow A = 45°$

Then from (1), $45° + B = 60° \Rightarrow B = 15°$

4. State whether the following are true or false. Justify your answer.

(i) $\sin(A + B) = \sin A + \sin B$

(ii) The value of $\sin\theta$ increases as θ increases.

(iii) The value of $\cos\theta$ increases as θ increases.

(iv) $\sin\theta = \cos\theta$ for all values of θ.

(v) cot A is not defined for $A = 0°$.

Sol. (i) False

When $A = 60°$, $B = 30°$

$LHS = \sin(A + B) = \sin(60° + 30°)$

$= \sin 90° = 1$

$RHS = \sin A + \sin B$

$= \sin 60° + \sin 30°$

$= \frac{\sqrt{3}}{2} + \frac{1}{2} \neq 1$

i.e., $LHS \neq RHS$

(ii) True

Note that $\sin 0° = 0$, $\sin 30° = \frac{1}{2} = 0.5$,

$\sin 45° = \frac{1}{\sqrt{2}} = 0.7$ (approx.),

$\sin 60° = \frac{\sqrt{3}}{2} = 0.87$ (approx.)

and $\sin 90° = 1$

i.e., value of $\sin\theta$ increases as θ increases from 0° to 90°.

(iii) False

Note that $\cos 0° = 1$,

$\cos 30° = \frac{\sqrt{3}}{2} = 0.87$ (approx.)

$\cos 45° = \frac{1}{\sqrt{2}} = 0.7$ (approx.),

$\cos 60° = \frac{1}{2} = 0.5$ and $\cos 90° = 0$

i.e., value of $\cos\theta$ decreases as θ increases from 0° to 90°.

(iv) False, it is true for only $\theta = 45°$

(v) True, $\cot A = \frac{1}{0}$ = not defined.

EXERCISE : 8.3

1. Evaluate :

(i) $\frac{\sin 18°}{\cos 72°}$

(ii) $\frac{\tan 26°}{\cot 64°}$

(iii) $\cos 48° - \sin 42°$

(iv) $\operatorname{cosec} 31° - \sec 59°$

Sol. (i) $\frac{\sin 18°}{\cos 72°} = \frac{\sin 18°}{\cos(90° - 18°)} = \frac{\sin 18°}{\sin 18°} = 1$

$\{\because \cos(90° - \theta) = \sin\theta\}$

(ii) $\frac{\tan 26°}{\cot 64°} = \frac{\tan(90° - 64°)}{\cot 64°} = \frac{\cot 64°}{\cot 64°} = 1$

(iii) $\cos 48° - \sin 42° = \cos(90° - 42°) - \sin 42°$

$= \sin 42° - \sin 42° = 0$

(iv) $\operatorname{cosec} 31° - \sec 59°$

$= \operatorname{cosec}(90° - 59°) - \sec 59°$

$= \sec 59° - \sec 59° = 0$

2. Show that

(i) $\tan 48° \tan 23° \tan 42° \tan 67° = 1$

(ii) $\cos 38° \cos 52° - \sin 38° \sin 52° = 0.$

Sol. (i) LHS = $\tan 48° \tan 23° \tan 42° \tan 67°$

$= \tan 48° \times \tan 23° \times \tan (90° - 48°) \times \tan (90° - 23°)$

$= \tan 48° \times \tan 23° \times \cot 48° \times \cot 23°$

$= \tan 48° \times \tan 23° \times \dfrac{1}{\tan 48°} \times \dfrac{1}{\tan 23°} = 1$

$\therefore$ LHS = RHS.

(ii) LHS = $\cos 38° \cos 52° - \sin 38° \sin 52°$

$= \cos (90° - 52°) \cos 52° - \sin (90° - 52°) \sin 52°$

$= \sin 52° \cos 52° - \cos 52° \sin 52° = 0$

3. If $\tan 2A = \cot (A - 18°)$, where 2A is an acute angle, find the value of A.

Sol. $\tan 2A = \cot (A - 18°)$

$\Rightarrow$ $\cot (90° - 2A) = \cot (A - 18°)$

$\Rightarrow$ $90° - 2A = A - 18°$

$\Rightarrow$ $108° = 3A$

$A = 36°$

4. If $\tan A = \cot B$, prove that $A + B = 90°$.

Sol. $\tan A = \cot B$

$\tan A = \tan (90° - B)$

$\therefore$ $A = 90° - B$

$A + B = 90°$

5. If $\sec 4A = \operatorname{cosec} (A - 20°)$, where 4 A is an acute angle, find the value of A.

Sol. $\sec 4A = \operatorname{cosec} (A - 20°)$

$\Rightarrow$ $\operatorname{cosec} (90° - 4A) = \operatorname{cosec} (A - 20°)$

$\{\because \operatorname{cosec} (90° - \theta) = \sec \theta\}$

$\Rightarrow$ $90° - 4A = A - 20°$

$\Rightarrow$ $5A = 110° \Rightarrow A = 22°$

6. If A, B and C are interior angles of a triangle ABC, then show that $\sin\left(\dfrac{B+C}{2}\right) = \cos \dfrac{A}{2}$.

Sol. $A + B + C = 180°$

$\Rightarrow$ $B + C = 180° - A$

$\Rightarrow$ $\dfrac{B+C}{2} = \dfrac{180° - A}{2}$

$\Rightarrow$ $\dfrac{B+C}{2} = \left(90° - \dfrac{A}{2}\right)$

$\Rightarrow$ $\sin\left(\dfrac{B+C}{2}\right) = \sin\left(90° - \dfrac{A}{2}\right) = \cos \dfrac{A}{2}$

$\{\because \sin (90° - \theta) = \cos \theta\}$

7. Express $\sin 67° + \cos 75°$ in terms of trigonometric ratios of angles between 0° and 45°.

Sol. $\sin 67° + \cos 75°$

$= \sin (90° - 23°) + \cos (90° - 15°)$

$= \cos 23° + \sin 15°$

EXERCISE : 8.4

1. Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

Sol. We have $\operatorname{cosec}^2 A - \cot^2 A = 1$

$\Rightarrow$ $\operatorname{cosec}^2 A = 1 + \cot^2 A$

$\Rightarrow$ $(\operatorname{cosec} A)^2 = \cot^2 A + 1$

$\Rightarrow$ $\left(\dfrac{1}{\sin A}\right)^2 = \cot^2 A + 1$

$\Rightarrow$ $(\sin A)^2 = \dfrac{1}{\cot^2 A + 1}$

$\Rightarrow$ $\sin A = \pm \dfrac{1}{\sqrt{\cot^2 A + 1}}$ We reject negative value of sin A for acute angle A. Therefore,

$\sin A = \tan A = \dfrac{1}{\cot A}$

We have $\sec^2 A - \tan^2 A = 1$

$\Rightarrow \quad \sec^2 A = 1 + \tan^2 A$

$= 1 + \dfrac{1}{\cot^2 A} = \dfrac{\cot^2 A + 1}{\cot^2 A}$

$\Rightarrow \quad \sec A = \dfrac{\sqrt{\cot^2 A + 1}}{\cot A}$

2. Write all the other trigonometric ratios of $\angle A$ in terms of sec A.

Sol. (i) $\sin A = \sqrt{1 - \cos^2 A}$

$= \sqrt{1 - \dfrac{1}{\sec^2 A}} = \dfrac{\sqrt{\sec^2 A - 1}}{\sec A}$

(ii) $\cos A = \dfrac{1}{\sec A}$

(iii) $\tan A = \sqrt{\sec^2 A - 1}$

(iv) $\cot A = \dfrac{1}{\tan A} = \dfrac{1}{\sqrt{\sec^2 A - 1}}$

(v) $\text{cosec } A = \dfrac{1}{\sin A} = \dfrac{\sec A}{\sqrt{\sec^2 A - 1}}$

3. Evaluate:

(i) $\dfrac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

(ii) sin 25° cos 65° + cos 25° sin 65°

Sol. (i) $\dfrac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

$= \dfrac{\{\sin(90^\circ - 27^\circ)\}^2 + \sin^2 27^\circ}{\cos^2 17^\circ + \{\cos(90^\circ - 17^\circ)\}^2}$

$= \dfrac{\{\cos 27^\circ\}^2 + \sin^2 27^\circ}{\cos^2 17^\circ + \{\sin 17^\circ\}^2}$

$= \dfrac{\cos^2 27^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \sin^2 17^\circ} = \dfrac{1}{1} = 1$

(ii) sin 25°cos 65° + cos 25°sin 65°

= sin (90° – 65°) cos 65° + cos (90°–65°) sin 65°

= cos 65° cos 65° + sin 65° sin 65°

$= \cos^2 65^\circ + \sin^2 65^\circ = 1$

4. Choose the correct option. Justify your choice :

(i) $9 \sec^2 A - 9 \tan^2 A =$

(A) 1 (B) 9

(C) 8 (D) 0

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\,\theta) =$

(A) 0 (B) 1

(C) 2 (D) – 1

(iii) (sec A + tan A)(1 – sin A) =

(A) sec A (B) sin A

(C) cosec A (D) cos A

(iv) $\dfrac{1 + \tan^2 A}{1 + \cot^2 A} =$

(A) $\sec^2 A$ (B) – 1

(C) $\cot^2 A$ (D) $\tan^2 A$

Sol. (i) Option (B)

$9 \sec^2 A - 9 \tan^2 A = 9(\sec^2 A - \tan^2 A)$

= 9 × 1 = 9.

(ii) Option (C)

$(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\,\theta)$

$= \left\{1 + \dfrac{\sin\theta}{\cos\theta} + \dfrac{1}{\cos\theta}\right\} \times \left\{1 + \dfrac{\cos\theta}{\sin\theta} - \dfrac{1}{\sin\theta}\right\}$

$= \left\{\dfrac{\cos\theta + \sin\theta + 1}{\cos\theta}\right\} \times \left\{\dfrac{\sin\theta + \cos\theta - 1}{\sin\theta}\right\}$

$= \dfrac{\{(\cos\theta + \sin\theta) + 1\} \times \{(\cos\theta + \sin\theta) - 1\}}{\cos\theta \times \sin\theta}$

$= \dfrac{(\cos\theta + \sin\theta)^2 - (1)^2}{\cos\theta \times \sin\theta}$

$\{\because (a + b)(a - b) = a^2 - b^2\}$

$= \dfrac{\cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta - 1}{\cos\theta \times \sin\theta}$

$= \dfrac{1 + 2\cos\theta\sin\theta - 1}{\cos\theta\sin\theta} = 2.$

(iii) Correct option is (D).

(sec A + tan A)(1 – sin A)

$= \sec A - \tan A + \tan A - \frac{\sin^2 A}{\cos A}$

$= \frac{1}{\cos A} - \frac{\sin^2 A}{\cos A} = \frac{1 - \sin^2 A}{\cos A}$

$= \frac{\cos^2 A}{\cos A} = \cos A$

(iv) Correct option is (D).

$\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{\sec^2 A}{\mathrm{cosec}^2 A} = \tan^2 A$

5. Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

(i) $(\mathrm{cosec}\,\theta - \cot\theta)^2 = \frac{1 - \cos\theta}{1 + \cos\theta}$.

(ii) $\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$.

(iii) $\frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta} = 1 + \sec\theta\,\mathrm{cosec}\,\theta$.

(iv) $\frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$.

(v) $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \mathrm{cosec}\,A + \cot A$, using the identity $\mathrm{cosec}^2 A = 1 + \cot^2 A$.

(vi) $\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$.

(vii) $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$.

(viii) $(\sin A + \mathrm{cosec}\,A)^2 + (\cos A + \sec A)^2$

$= 7 + \tan^2 A + \cot^2 A$.

(ix) $(\mathrm{cosec}\,A - \sin A)(\sec A - \cos A)$

$= \frac{1}{\tan A + \cot A}$.

(x) $\left(\frac{1 + \tan^2 A}{1 + \cot^2 A}\right) = \left(\frac{1 - \tan A}{1 - \cot A}\right)^2 = \tan^2 A$.

Sol. (i) LHS = $(\mathrm{cosec}\,\theta - \cot\theta)^2$

$= \left\{\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right\}^2 = \left(\frac{1 - \cos\theta}{\sin\theta}\right)^2$

$= \frac{(1 - \cos\theta)^2}{\sin^2\theta} = \frac{(1 - \cos\theta)^2}{1 - \cos^2\theta}$

$= \frac{(1 - \cos\theta)^2}{(1 - \cos\theta) \times (1 + \cos\theta)} = \frac{1 - \cos\theta}{1 + \cos\theta}$

∴ LHS = RHS.

(ii) LHS = $\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A}$

$= \frac{\cos^2 A + (1 + \sin A)^2}{\cos A(1 + \sin A)}$

$= \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{\cos A(1 + \sin A)}$

$= \frac{2(1 + \sin A)}{\cos A(1 + \sin A)}$

= 2 sec A = R.H.S.

(iii) LHS = $\frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta}$

$= \frac{\left(\frac{\sin\theta}{\cos\theta}\right)}{\left(1 - \frac{\cos\theta}{\sin\theta}\right)} + \frac{\left(\frac{\cos\theta}{\sin\theta}\right)}{\left(1 - \frac{\sin\theta}{\cos\theta}\right)}$

$= \frac{\left(\frac{\sin\theta}{\cos\theta}\right)}{\left(1 - \frac{\cos\theta}{\sin\theta}\right)} + \frac{\left(\frac{\cos\theta}{\sin\theta}\right)}{\left(1 - \frac{\sin\theta}{\cos\theta}\right)}$

$= \frac{\sin\theta \times \sin\theta}{\cos\theta \times (\sin\theta - \cos\theta)} + \frac{\sin\theta \times \sin\theta}{\cos\theta \times (\sin\theta - \cos\theta)}$

$= \frac{\sin^2\theta}{\cos\theta \times (\sin\theta - \cos\theta)} - \frac{\cos^2\theta}{\sin\theta \times (\sin\theta - \cos\theta)}$

$= \frac{\sin\theta \times \sin^2\theta - \cos\theta \times \cos^2\theta}{\cos\theta \times \sin\theta \times (\sin\theta - \cos\theta)}$

$$= \frac{\sin^3\theta - \cos^3\theta}{\cos\theta \times \sin\theta \times (\sin\theta - \cos\theta)}$$

$$= \frac{(\sin\theta - \cos\theta)\times(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)}{\cos\theta\times\sin\theta\times(\sin\theta - \cos\theta)}$$

$\{\because a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$$= \frac{\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta}{\cos\theta\times\sin\theta}$$

$$= \frac{1 + \sin\theta\cos\theta}{\cos\theta\sin\theta} = \frac{1}{\cos\theta\sin\theta} + 1$$

$$= 1 + \left(\frac{1}{\cos\theta}\right)\left(\frac{1}{\sin\theta}\right)$$

$= 1 + \sec\theta \operatorname{cosec}\theta$

∴ LHS = RHS

(iv) $\text{L.H.S.} = \frac{1 + \sec A}{\sec A} = \frac{1 + \frac{1}{\cos A}}{\frac{1}{\cos A}} = \frac{\cos A + 1}{1}$

$\text{R.H.S.} = \frac{\sin^2 A}{1 - \cos A} = \frac{1 - \cos^2 A}{1 - \cos A} = 1 + \cos A$

∴ L.H.S. = R.H.S.

(v) $\text{LHS} = \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}}$$

(Dividing the numerator and denominator by sin A)

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A} = \frac{(\operatorname{cosec} A + \cot A) - 1}{\{1 + \cot A - \operatorname{cosec} A\}}$$

$$= \frac{(\operatorname{cosec} A + \cot A) - (\operatorname{cosec}^2 A - \cot^2 A)}{\{1 + \cot A - \operatorname{cosec} A\}}$$

$(\because \operatorname{cosec}^2 A = 1 + \cot^2 A, \text{i.e.,} \operatorname{cosec}^2 A - \cot^2 A = 1)$

$$\frac{(\operatorname{cosec} A + \cot A) - (\operatorname{cosec} A + \cot A) \times (\operatorname{cosec} A - \cot A)}{\{1 + \cot A - \operatorname{cosec} A\}}$$

$\{\because (a + b)(a - b) = a^2 - b^2\}$

$$= \frac{(\operatorname{cosec} A + \cot A) \times \{1 - (\operatorname{cosec} A - \cot A)\}}{\{1 + \cot A - \operatorname{cosec} A\}}$$

$$= \frac{(\operatorname{cosec} A + \cot A) \times \{1 + \cot A - \operatorname{cosec} A\}}{\{1 + \cot A - \operatorname{cosec} A\}}$$

$= \operatorname{cosec} A + \cot A$

= RHS

(vi) $\text{LHS} = \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sqrt{\frac{(1 + \sin A)(1 + \sin A)}{(1 - \sin A)(1 + \sin A)}}$

$$= \sqrt{\frac{(1 + \sin A)^2}{(1)^2 - (\sin A)^2}} = \sqrt{\frac{(1 + \sin A)^2}{1 - \sin^2 A}}$$

$$= \sqrt{\frac{(1 + \sin A)^2}{\cos^2 A}} = \frac{1 + \sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A$$

∴ LHS = RHS.

(vii) $\text{L.H.S.} = \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$$= \frac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta(\sin^2\theta + \cos^2\theta - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - \sin^2\theta - \cos^2\theta)}$$

$$= \frac{\tan\theta(\cos^2\theta - \sin^2\theta)}{(\cos^2\theta - \sin^2\theta)}$$

$= \tan\theta$ = R.H.S.

(viii) $\text{L.H.S.} = (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \operatorname{cosec}^2 A + 2 + \cos^2 A + \sec^2 A + 2$

$= 4 + 1 + 1 + \cot^2 A + 1 + \tan^2 A$

$= 7 + \tan^2 A + \cot^2 A$ = R.H.S.

(ix) LHS = (cosec A – sin A) (sec A – cos A)

$$= \left(\frac{1}{\sin A} - \sin A\right) \times \left(\frac{1}{\cos A} - \cos A\right)$$

$$= \frac{1-\sin^2 A}{\sin A} \times \frac{1-\cos^2 A}{\cos A}$$

$$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A} = \sin A \cos A$$

Now, RHS $= \dfrac{1}{\tan A + \cot A} = \dfrac{1}{\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}}$

$$= \frac{1}{\dfrac{\sin^2 A + \cos^2 A}{\sin A \cos A}} = \frac{\sin A \cos A}{\sin^2 A + \cos^2 A}$$

$$= \frac{\sin A \cos A}{1}$$

∴ LHS = RHS.

(x) L.H.S. $= \left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right) = \dfrac{\sec^2 A}{\operatorname{cosec}^2 A}$

$= \tan^2 A$ = R.H.S.

& $\left(\dfrac{1-\tan A}{1-\cot A}\right)^2 = \left(\dfrac{\dfrac{\cos A - \sin A}{\cos A}}{\dfrac{\sin A - \cos A}{\sin A}}\right)^2$

$= \dfrac{\sin^2 A}{\cos^2 A} = \tan^2 A$ = R.H.S.