

## NCERT QUESTIONS WITH SOLUTIONS

### EXERCISE : 4.1

1. Check whether the following are quadratic equations :

(i)  $(x + 1)^2 = 2(x - 3)$

(ii)  $x^2 - 2x = (-2)(3 - x)$

(iii)  $(x - 2)(x + 1) = (x - 1)(x + 3)$

(iv)  $(x - 3)(2x + 1) = x(x + 5)$

(v)  $(2x - 1)(x - 3) = (x + 5)(x - 1)$

(vi)  $x^2 + 3x + 1 = (x - 2)^2$

(vii)  $(x + 2)^3 = 2x(x^2 - 1)$

(viii)  $x^3 - 4x^2 - x + 1 = (x - 2)^3$

**Sol.** (i)  $(x + 1)^2 = 2(x - 3)$

$$\Rightarrow x^2 + 2x + 1 = 2x - 6$$

$$\Rightarrow x^2 + 2x - 2x + 1 + 6 = 0$$

$$\Rightarrow x^2 + 0x + 7 = 0$$

It is of the form  $ax^2 + bx + c = 0$ .

Hence, the given equation is a quadratic equation.

(ii)  $x^2 - 2x = (-2)(3 - x)$

$$\Rightarrow x^2 - 2x = -6 + 2x$$

$$\Rightarrow x^2 - 4x + 6 = 0$$

It is of the form  $ax^2 + bx + c = 0$ .

Hence, the given equation is a quadratic equation.

(iii)  $(x - 2)(x + 1) = (x - 1)(x + 3)$

$$\Rightarrow x^2 + x - 2x - 2 = x^2 + 3x - x - 3$$

$$\Rightarrow x^2 - x - 2 = x^2 + 2x - 3$$

$$\Rightarrow -x - 2x - 2 + 3 = 0$$

$$\Rightarrow -3x + 1 = 0 \text{ or } 3x - 1 = 0$$

It is not of the form  $ax^2 + bx + c = 0$ .

Hence, the given equation is not a quadratic equation.

(iv)  $(x - 3)(2x + 1) = x(x + 5)$

$$\Rightarrow 2x^2 - 5x - 3 = x^2 + 5x$$

$$\Rightarrow x^2 - 10x - 3 = 0$$

It is of the form  $ax^2 + bx + c = 0$ .

Hence, the given equation is a quadratic equation.

(v)  $(2x - 1)(x - 3) = (x + 5)(x - 1)$

$$\Rightarrow 2x^2 - 7x + 3 = x^2 + 4x - 5$$

$$\Rightarrow x^2 - 11x + 8 = 0$$

It is of the form  $ax^2 + bx + c = 0$ .

Hence, the given equation is a quadratic equation.

(vi)  $x^2 + 3x + 1 = (x - 2)^2$

$$\Rightarrow x^2 + 3x + 1 = x^2 + 4 - 4x$$

$$\Rightarrow 7x - 3 = 0$$

It is not of the form  $ax^2 + bx + c = 0$ .

Hence, the given equation is not a quadratic equation.

(vii)  $(x + 2)^3 = 2x(x^2 - 1)$

$$\Rightarrow x^3 + 3 \times x \times 2 \times (x + 2) + 2^3$$

$$= 2x(x^2 - 1)$$

$$\Rightarrow x^3 + 6x(x + 2) + 8 = 2x^3 - 2x$$

$$\Rightarrow x^3 + 6x^2 + 12x + 8 = 2x^3 - 2x$$

$$\Rightarrow -x^3 + 6x^2 + 14x + 8 = 0$$

$$\Rightarrow x^3 - 6x^2 - 14x - 8 = 0$$

It is not of the form  $ax^2 + bx + c = 0$ .

Hence, the given equation is not a quadratic equation.

(viii)  $x^3 - 4x^2 - x + 1 = (x - 2)^3$

$$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 8 - 6x^2 + 12x$$

$$\Rightarrow 2x^2 - 13x + 9 = 0$$

It is of the form  $ax^2 + bx + c = 0$ .

Hence, the given equation is a quadratic equation.

2. Represent the following situations in the form of quadratic equations :

- (i) The area of a rectangular plot is 528 m<sup>2</sup>. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.
- (ii) The product of two consecutive positive integers is 306. We need to find the integers.
- (iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.
- (iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

**Sol.** (i) Let breadth = x meters  
Then length = (2x + 1) meters  
⇒ Area of the plot = x × (2x + 1) = 528  
or 2x<sup>2</sup> + x - 528 = 0

(ii) Let the two consecutive positive integers be x and x + 1. It is given that their product is 306.  
∴ x(x + 1) = 306  
⇒ x<sup>2</sup> + x - 306 = 0

(iii) Let Rohan's present age = x years  
Then present age of Rohan's mother = (x + 26) years  
After 3 years, it is given that  
(x + 3) × [(x + 26) + 3] = 360

⇒ (x + 3)(x + 29) = 360  
⇒ x<sup>2</sup> + 32x + 87 = 360  
⇒ x<sup>2</sup> + 32x + 87 - 360 = 0  
⇒ x<sup>2</sup> + 32x - 273 = 0

(iv) Let the speed of train be x km/h.  
Time taken to travel 480 km =  $\frac{480}{x}$  h.  
In second condition,  
the speed of train = (x - 8) km/h  
It is also given that the train will take 3 hours more to cover the same distance. Therefore, time taken to travel 480 km =  $\left(\frac{480}{x} + 3\right)$  h  
Speed × Time = Distance  
(x - 8)  $\left(\frac{480}{x} + 3\right)$  = 480  
⇒ 480 + 3x -  $\frac{3840}{x}$  - 24 = 480  
⇒ 3x -  $\frac{3840}{x}$  = 24  
⇒ 3x<sup>2</sup> - 24x - 3840 = 0  
⇒ x<sup>2</sup> - 8x - 1280 = 0

**EXERCISE : 4.2**

1. Find the roots of the following quadratic equations by factorisation :
- (i) x<sup>2</sup> - 3x - 10 = 0
  - (ii) 2x<sup>2</sup> + x - 6 = 0
  - (iii)  $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$
  - (iv) 2x<sup>2</sup> - x +  $\frac{1}{8} = 0$
  - (v) 100x<sup>2</sup> - 20x + 1 = 0

**Sol.** (i)  $x^2 - 3x - 10 = 0$   
 $\Rightarrow x^2 - 5x + 2x - 10 = 0$   
 $\Rightarrow x(x - 5) + 2(x - 5) = 0$   
 $\Rightarrow (x + 2)(x - 5) = 0$   
 $\Rightarrow x + 2 = 0$  or  $x - 5 = 0$   
 $\Rightarrow x = -2$  or  $x = 5$   
 Hence, the roots are  $-2$  and  $5$ .

(ii)  $2x^2 + x - 6 = 0$   
 $\Rightarrow 2x^2 + 4x - 3x - 6 = 0$   
 $\Rightarrow 2x(x + 2) - 3(x + 2) = 0$   
 $\Rightarrow (x + 2)(2x - 3) = 0$   
 $\Rightarrow x + 2 = 0$   
 or  $2x - 3 = 0$   
 $\Rightarrow x = -2$  or  $x = \frac{3}{2}$

(iii)  $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$   
 $\Rightarrow \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$   
 $\Rightarrow x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$   
 $\Rightarrow (x + \sqrt{2})(\sqrt{2}x + 5) = 0$   
 $\Rightarrow x = -\sqrt{2}$  or  $x = -\frac{5}{\sqrt{2}}$   
 Hence, the roots are  $-\sqrt{2}$  and  $-\frac{5}{\sqrt{2}}$

(iv)  $2x^2 - x + \frac{1}{8} = 0$   
 $\Rightarrow 16x^2 - 8x + 1 = 0$   
 $\Rightarrow (4x)^2 - 2 \times 4x \times 1 + 1^2 = 0$   
 $\Rightarrow (4x - 1)^2 = 0$   
 $\Rightarrow$  Both roots are given by  $4x - 1 = 0$ ,  
 i.e.  $x = \frac{1}{4}$ . Hence, the roots are  $\frac{1}{4}, \frac{1}{4}$ .

(v)  $100x^2 - 20x + 1 = 0$   
 $\Rightarrow (10x)^2 - 2 \times 10x \times 1 + 1^2 = 0$   
 $\Rightarrow (10x - 1)^2 = 0$   
 $\Rightarrow$  Both roots are given by  $10x - 1 = 0$ ,  
 i.e.  $x = \frac{1}{10}$ . Hence, the roots are  $\frac{1}{10}, \frac{1}{10}$

- 2.** Solve the following problems :
- (i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.
- (ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ₹ 750. We would like to find out the number of toys produced on that day.

**Sol.** (i) Let the number of marbles John had be  $x$ . Then the number of marbles Jivanti had =  $45 - x$   
 The number of marbles left with John, when he lost 5 marbles =  $x - 5$   
 The number of marbles left with Jivanti, when she lost 5 marbles =  $45 - x - 5 = 40 - x$   
 Therefore, their product =  $(x - 5)(40 - x)$   
 $= 40x - x^2 - 200 + 5x$   
 $= -x^2 + 45x - 200$   
 So,  $-x^2 + 45x - 200 = 124$   
 (Given that product = 124)

$$\begin{aligned} \Rightarrow -x^2 + 45x - 324 &= 0 \\ \Rightarrow x^2 - 45x + 324 &= 0 \\ \Rightarrow x^2 - 36x - 9x + 324 &= 0 \\ \Rightarrow x(x - 36) - 9(x - 36) &= 0 \\ \Rightarrow (x - 9)(x - 36) &= 0 \\ \Rightarrow x - 9 = 0 \text{ or } x - 36 &= 0 \\ \Rightarrow x = 9 \text{ or } x = 36 \\ \Rightarrow 45 - x = 36 \text{ or } 45 - x &= 9 \end{aligned}$$

Therefore, the number of marbles John and Jivanti had 9 and 36 respectively or 36 and 9 respectively.

(ii) Let the number of toys produced be  $x$ .  
 $\therefore$  Cost of production of each toy = ₹  $(55 - x)$   
 It is given that, total cost of production of the toys = ₹ 750  
 $\therefore x(55 - x) = 750$   
 $\Rightarrow x^2 - 55x + 750 = 0$   
 $\Rightarrow x^2 - 30x - 25x + 750 = 0$   
 $\Rightarrow x(x - 30) - 25(x - 30) = 0$   
 $\Rightarrow (x - 25)(x - 30) = 0$   
 $\Rightarrow x - 25 = 0 \text{ or } x - 30 = 0$   
 $\Rightarrow x = 25 \text{ or } x = 30$   
 $\Rightarrow 55 - x = 30 \text{ or } 55 - x = 25$

Therefore, the number of toys produced are either 25 or 30.

3. Find two numbers whose sum is 27 and product is 182.

**Sol.** Let one number be  $x$ , then second number =  $27 - x$   
 Given that  $x \times (27 - x) = 182$   
 $\Rightarrow 27x - x^2 = 182$   
 $\Rightarrow x^2 - 27x + 182 = 0$   
 $\Rightarrow x^2 - 14x - 13x + 182 = 0$   
 $\Rightarrow x(x - 14) - 13(x - 14) = 0$   
 $\Rightarrow (x - 13)(x - 14) = 0$

Either  $x - 13 = 0$  or  $x - 14 = 0$ ,  
 i.e.  $x = 13$  or  $14$

$\Rightarrow 27 - x = 14$  or  $13$

Hence, the two marbles are 13 and 14.

4. Find two consecutive positive integers, sum of whose squares is 365.

**Sol.** Let the consecutive positive integers be  $x$  and  $(x + 1)$ .

Given that  $x^2 + (x + 1)^2 = 365$

$$\begin{aligned} \Rightarrow x^2 + x^2 + 1 + 2x &= 365 \\ \Rightarrow 2x^2 + 2x - 364 &= 0 \\ \Rightarrow x^2 + x - 182 &= 0 \\ \Rightarrow x^2 + 14x - 13x - 182 &= 0 \\ \Rightarrow x(x + 14) - 13(x + 14) &= 0 \\ \Rightarrow (x + 14)(x - 13) &= 0 \end{aligned}$$

Either  $x + 14 = 0$  or  $x - 13 = 0$ ,  
 i.e.  $x = -14$  or  $x = 13$

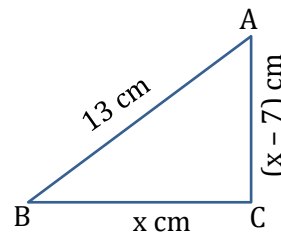
Since the integers are positive,  $x$  can only be 13.

$\therefore x + 1 = 13 + 1 = 14$

Therefore, two consecutive positive integers will be 13 and 14.

5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

**Sol.**



In  $\triangle ABC$ , base  $BC = x$  cm,  
 altitude  $AC = (x - 7)$  cm,  
 $\angle ACB = 90^\circ$  and  
 $AB = 13$  cm

By Pythagoras theorem,  
we have

$$BC^2 + AC^2 = AB^2$$

$$\Rightarrow x^2 + (x - 7)^2 = 13^2$$

$$\Rightarrow x^2 + x^2 - 14x + 49 = 169$$

$$\Rightarrow 2x^2 - 14x - 120 = 0$$

$$\Rightarrow x^2 - 7x - 60 = 0$$

$$\Rightarrow x^2 - 12x + 5x - 60 = 0$$

$$\Rightarrow x(x - 12) + 5(x - 12) = 0$$

$$\Rightarrow (x + 5)(x - 12) = 0$$

i.e.  $x = -5$  or  $x = 12$

We reject  $x = -5$ , because length can't be negative.

$$\therefore x = 12$$

Therefore,  $BC = 12$  cm and  $AC = 5$  cm.

6. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ₹ 90, find the number of articles produced and the cost of each article.

**Sol.** Let the number of articles produced be  $x$ .  
Therefore, cost of production of each article = ₹  $(2x + 3)$   
It is given that the total cost of production is ₹ 90.

$$\therefore x(2x + 3) = 90$$

$$\Rightarrow 2x^2 + 3x - 90 = 0$$

$$\Rightarrow 2x^2 + 15x - 12x - 90 = 0$$

$$\Rightarrow x(2x + 15) - 6(2x + 15) = 0$$

$$\Rightarrow (2x + 15)(x - 6) = 0$$

Either  $2x + 15 = 0$  or  $x - 6 = 0$ ,

i.e.,  $x = \frac{-15}{2}$  or  $x = 6$

As the number of articles produced can only be a positive integer, therefore,  $x$  can only be 6.

Hence, number of articles produced = 6

Cost of each article =  $2 \times 6 + 3 = ₹ 15$

**EXERCISE : 4.3**

1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them :

(i)  $2x^2 - 3x + 5 = 0$

(ii)  $3x^2 - 4\sqrt{3}x + 4 = 0$

(iii)  $2x^2 - 6x + 3 = 0$

- Sol.** (i)  $2x^2 - 3x + 5 = 0$

Here  $a = 2$ ,  $b = -3$  and  $c = 5$

Therefore, discriminant  $D = b^2 - 4ac$   
 $= (-3)^2 - 4 \times 2 \times 5$   
 $= 9 - 40 = -31$

As  $D < 0$

Hence, no real root.

(ii)  $3x^2 - 4\sqrt{3}x + 4 = 0$

Here  $a = 3$ ,  $b = -4\sqrt{3}$  and  $c = 4$

Therefore, discriminant  $D = b^2 - 4ac$   
 $= (-4\sqrt{3})^2 - 4(3)(4) = 48 - 48 = 0$

As  $D = 0$

Hence, two equal real roots.

Now, the roots are

$$= \frac{-b \pm \sqrt{D}}{2a} = \frac{4\sqrt{3} \pm 0}{2 \times 3} = \frac{2}{\sqrt{3}}$$

Hence, the roots are  $\frac{2}{\sqrt{3}}$  and  $\frac{2}{\sqrt{3}}$ .

(iii)  $2x^2 - 6x + 3 = 0$   
 $a = 2, b = -6$  and  $c = 3$   
 Discriminant  $D = b^2 - 4ac$   
 $= (-6)^2 - 4(2)(3)$   
 $= 36 - 24 = 12$   
 $\Rightarrow D > 0$   
 Hence, roots are distinct and real.  
 The roots are

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{6 \pm \sqrt{12}}{2 \times 2} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}$$

Therefore, the roots are

$$\frac{3 + \sqrt{3}}{2} \text{ and } \frac{3 - \sqrt{3}}{2}.$$

2. Find the values of  $k$  for each of the following quadratic equations, so that they have two equal roots.

(i)  $2x^2 + kx + 3 = 0$

(ii)  $kx(x - 2) + 6 = 0$

**Sol.** (i)  $2x^2 + kx + 3 = 0$

Here  $a = 2, b = k$  and  $c = 3$

Therefore,  $D = b^2 - 4ac$

$$= k^2 - 4 \times 2 \times 3 = k^2 - 24$$

Two roots will be equal if  $D = 0$

$$\Rightarrow k^2 - 24 = 0$$

$$\Rightarrow k = \pm \sqrt{24}$$

$$\Rightarrow k = \pm 2\sqrt{6}$$

(ii)  $kx(x - 2) + 6 = 0$

$$\text{or } kx^2 - 2kx + 6 = 0$$

Here,  $a = k, b = -2k$  and  $c = 6$

Therefore,  $D = b^2 - 4ac$

$$= (-2k)^2 - 4(k)(6) = 4k^2 - 24k$$

Two roots will be equal if  $D = 0 =$

$$\Rightarrow 4k^2 - 24k = 0$$

$$\Rightarrow 4k(k - 6) = 0$$

Either  $4k = 0$  or  $k - 6 = 0$

$$k = 0 \text{ or } k = 6$$

However, if  $k = 0$ , then the equation will not have the terms ' $x^2$ ' and ' $x$ '.

Hence,  $k = 6$ .

3. Is it possible to design a rectangular mango grove whose length is twice its breadth, and area is  $800 \text{ m}^2$ ? If so, find its length and breadth.

**Sol.** Let  $x$  be the breadth and  $2x$  be the length of the rectangle.

$$x \times 2x = 800$$

$$\Rightarrow 2x^2 = 800$$

$$\Rightarrow x^2 = 400 = (20)^2$$

$$\Rightarrow x = 20$$

Hence, the rectangle is possible and it has breadth =  $20 \text{ m}$  and length =  $40 \text{ m}$ .

4. Is the following situation possible? If so, determine their present ages.

The sum of the ages of two friends is  $20$  years. Four years ago, the product of their ages in years was  $48$ .

**Sol.** Let the age of one friend be  $x$  years.

Age of the other friend will be  $(20 - x)$  years.

4 years ago, age of first friend =  $(x - 4)$  years and age of second friend =  $(20 - x - 4)$  =  $(16 - x)$  years

Given that,

$$(x - 4)(16 - x) = 48$$

$$16x - 64 - x^2 + 4x = 48$$

$$-x^2 + 20x - 112 = 0$$

$$x^2 - 20x + 112 = 0$$

Here,  $a = 1$ ,  $b = -20$  and  $c = 112$

Therefore,  $D = b^2 - 4ac$

$$= (-20)^2 - 4(1)(112)$$

$$= 400 - 448 = -48$$

As  $D < 0$ ,

Therefore, no real root is possible for this equation and hence, this situation is not possible.

5. Is it possible to design a rectangular park of perimeter 80 m and area  $400 \text{ m}^2$ ? If so, find its length and breadth.

**Sol.** Perimeter of the rectangular park = 80 m

$$\Rightarrow \text{Length} + \text{Breadth of the park} = \frac{80}{2}$$

$$m = 40 \text{ m.}$$

Let the breadth be  $x$  metres, then

$$\text{length} = (40 - x) \text{ m.}$$

$$\text{Here, } x < 40.$$

$$x \times (40 - x) = 400$$

[Each = area of the park]

$$\Rightarrow -x^2 + 40x - 400 = 0$$

$$\Rightarrow x^2 - 40x + 400 = 0$$

$$\Rightarrow (x - 20)^2 = 0$$

$$\Rightarrow x = 20$$

Thus, we have length = breadth = 20 m

Therefore, the park is in a shape of a square having 20 m side.

**Deleted Exercise**

1. Find the roots of the given quadratic equations by applying the quadratic formula.

(i)  $2x^2 - 7x + 3 = 0$

(ii)  $2x^2 + x - 4 = 0$

(iii)  $4x^2 + 4\sqrt{3}x + 3 = 0$

(iv)  $2x^2 + x + 4 = 0$

- Sol.** (i)  $2x^2 - 7x + 3 = 0$

$$a = 2, b = -7 \text{ and } c = 3$$

$$D = (-7)^2 - 4(2)(3) = 25$$

$$\Rightarrow \sqrt{D} = \sqrt{25} = 5$$

Roots of the given quadratic

equation are  $\frac{-b \pm \sqrt{D}}{2a}$ ,

i.e.,  $\frac{7 \pm 5}{2 \times 2}$ , i.e. the roots are 3 and  $\frac{1}{2}$ .

(ii)  $2x^2 + x - 4 = 0$

$$a = 2, b = 1 \text{ and } c = -4$$

$$D = (1)^2 - 4(2)(-4) = 33$$

$$\Rightarrow \sqrt{D} = \sqrt{33}$$

Roots are given by

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{33}}{4}$$

Hence, the two roots of the quadratic

equation are  $\frac{-1 + \sqrt{33}}{4}$  and  $\frac{-1 - \sqrt{33}}{4}$ .

(iii)  $4x^2 + 4\sqrt{3}x + 3 = 0$

$$a = 4, b = 4\sqrt{3} \text{ and } c = 3$$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-4\sqrt{3} \pm \sqrt{48 - 48}}{8}$$

$$\Rightarrow x = \frac{-4\sqrt{3} \pm 0}{8} = \frac{-\sqrt{3}}{2}$$

Hence, the roots are  $\frac{-\sqrt{3}}{2}$  and  $\frac{-\sqrt{3}}{2}$ .

(iv)  $2x^2 + x + 4 = 0$

$a = 2, b = 1$  and  $c = 4$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1 - 32}}{4}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{-31}}{4}$$

However, the square of a number cannot be negative. Therefore, there is no real root for the given equation.

2. Find the roots of the following equations :

(i)  $x - \frac{1}{x} = 3, x \neq 0$

(ii)  $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

**Sol.** (i)  $x - \frac{1}{x} = 3 \Rightarrow x^2 - 3x - 1 = 0$

On comparing this equation with

$$ax^2 + bx + c = 0,$$

we obtain  $a = 1, b = -3$  and  $c = -1$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{3 \pm \sqrt{9 + 4}}{2}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{13}}{2}$$

Therefore,  $x = \frac{3 + \sqrt{13}}{2}$  or  $\frac{3 - \sqrt{13}}{2}$

(ii)  $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

$$\Rightarrow \frac{(x-7) - (x+4)}{(x+4)(x-7)} = \frac{11}{30}$$

$$\Rightarrow \frac{-11}{(x^2 - 3x - 28)} = \frac{11}{30}$$

$$\Rightarrow -(x^2 - 3x - 28) = 30$$

$$\Rightarrow x^2 - 3x - 28 + 30 = 0$$

$$\Rightarrow x^2 - 3x + 2 = 0$$

$$\Rightarrow x^2 - 2x - x + 2 = 0$$

$$\Rightarrow x(x-2) - 1(x-2) = 0$$

$$\Rightarrow (x-2)(x-1) = 0$$

Hence, the two roots are 1 and 2.

3. Find the roots of the following quadratic equations, if they exist, by the method of completing the square:

(i)  $2x^2 - 7x + 3 = 0$

(ii)  $2x^2 + x - 4 = 0$

(iii)  $4x^2 + 4\sqrt{3}x + 3 = 0$

(iv)  $2x^2 + x + 4 = 0$

**Sol.** (i)  $2x^2 - 7x + 3 = 0$

$$\Rightarrow x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

$$\Rightarrow \left[ x^2 - 2 \times \frac{7}{4}x + \left( \frac{7}{4} \right)^2 \right] - \left( \frac{7}{4} \right)^2 + \frac{3}{2} = 0$$

$$\Rightarrow \left( x - \frac{7}{4} \right)^2 - \frac{49}{16} + \frac{3}{2} = 0$$

$$\Rightarrow \left( x - \frac{7}{4} \right)^2 - \frac{25}{16} = 0$$

$$\Rightarrow \left( x - \frac{7}{4} \right)^2 = \frac{25}{16}$$

$$\Rightarrow x - \frac{7}{4} = \pm \sqrt{\frac{25}{16}}$$

$$\Rightarrow x - \frac{7}{4} = \pm \frac{5}{4}$$

$$\Rightarrow x - \frac{7}{4} = \frac{5}{4} \quad \text{or} \quad x - \frac{7}{4} = -\frac{5}{4}$$

$$\Rightarrow x = \frac{5}{4} + \frac{7}{4} \quad \text{or} \quad x = -\frac{5}{4} + \frac{7}{4}$$

$$\Rightarrow x = 3 \quad \text{or} \quad x = \frac{1}{2}$$

Hence, the roots of the given quadratic equation are  $\frac{1}{2}$  and 3.

$$(ii) \quad 2x^2 + x - 4 = 0$$

$$\Rightarrow 2x^2 + x = 4$$

On dividing both sides of the equation by 2, we obtain

$$\Rightarrow x^2 + \frac{1}{2}x = 2$$

On adding  $\left(\frac{1}{4}\right)^2$  to both sides of the equation, we obtain

$$\Rightarrow (x)^2 + 2 \times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 = 2 + \left(\frac{1}{4}\right)^2$$

$$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{33}{16}$$

$$\Rightarrow x + \frac{1}{4} = \pm \frac{\sqrt{33}}{4}$$

$$\Rightarrow x = \frac{\pm\sqrt{33}}{4} - \frac{1}{4}$$

$$\Rightarrow x = \frac{\sqrt{33}-1}{4} \quad \text{or} \quad \frac{-\sqrt{33}-1}{4}$$

$$(iii) \quad 4x^2 + 4\sqrt{3}x + 3 = 0$$

$$\Rightarrow (2x)^2 + 2 \times 2x \times \sqrt{3} + (\sqrt{3})^2 = 0$$

$$\Rightarrow (2x + \sqrt{3})^2 = 0$$

$$\Rightarrow (2x + \sqrt{3}) = 0 \quad \text{or} \quad (2x + \sqrt{3}) = 0$$

$$\Rightarrow x = \frac{-\sqrt{3}}{2} \quad \text{or} \quad x = \frac{-\sqrt{3}}{2}$$

$$(iv) \quad 2x^2 + x + 4 = 0$$

$$\Rightarrow 2x^2 + x = -4$$

On dividing both sides of the equation by 2, we obtain

$$\Rightarrow x^2 + \frac{1}{2}x = -2$$

$$\Rightarrow x^2 + 2 \times x \times \frac{1}{4} = -2$$

On adding  $\left(\frac{1}{4}\right)^2$  to both sides of the equation, we obtain

$$\Rightarrow (x)^2 + 2 \times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 = \left(\frac{1}{4}\right)^2 - 2$$

$$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{1}{16} - 2$$

$$\Rightarrow \left(x + \frac{1}{4}\right)^2 = -\frac{31}{16}$$

However, the square of a number cannot be negative. Therefore, there is no real root for the given equation.

4. The sum of the reciprocals of Rehman's ages (in years) 3 years ago and 5 years from now is  $\frac{1}{3}$ . Find his present age.

**Sol.** Let the present age of Rehman be  $x$  years.

$$\text{We are given that } \frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

$$\Rightarrow \frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3}$$

$$\Rightarrow 3(2x+2) = (x-3)(x+5)$$

$$\Rightarrow 6x+6 = x^2+2x-15$$

$$\Rightarrow x^2-4x-21=0$$

$$\Rightarrow x^2-7x+3x-21=0$$

$$\Rightarrow x(x-7)+3(x-7)=0$$

$$\Rightarrow (x-7)(x+3)=0$$

We reject  $x = -3$

( $\because$   $x$  cannot be negative)

$$\therefore x = 7$$

Hence, Rehman's present age = 7 years.

5. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

**Sol.** Let the marks in Maths be  $x$ .

Then, the marks in English will be  $30 - x$ .

According to the given question,

$$(x + 2)(30 - x - 3) = 210$$

$$(x + 2)(27 - x) = 210$$

$$\Rightarrow -x^2 + 25x + 54 = 210$$

$$\Rightarrow x^2 - 25x + 156 = 0$$

$$\Rightarrow x^2 - 12x - 13x + 156 = 0$$

$$\Rightarrow x(x - 12) - 13(x - 12) = 0$$

$$\Rightarrow (x - 12)(x - 13) = 0$$

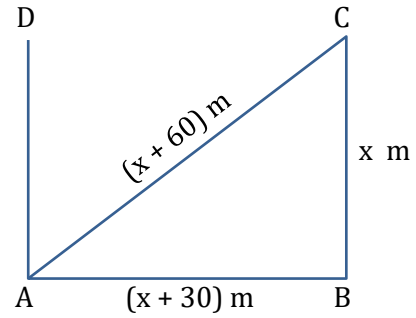
$$\Rightarrow x = 12 \text{ or } 13$$

If the marks in Maths are 12, then marks in English will be  $30 - 12 = 18$ .

If the marks in Maths are 13, then marks in English will be  $30 - 13 = 17$

6. The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

**Sol.**



In rectangle ABCD, let the shorter side  $BC = x$  metres.

Then  $AB = (x + 30)$  metres and diagonal  $AC = (x + 60)$  metres.

By Pythagoras Theorem we have

$$AB^2 + BC^2 = AC^2$$

$$\Rightarrow (x + 30)^2 + x^2 = (x + 60)^2$$

$$\Rightarrow x^2 + 60x + 900 + x^2$$

$$= x^2 + 120x + 3600$$

$$\Rightarrow x^2 - 60x - 2700 = 0$$

$$\Rightarrow x^2 - 90x + 30x - 2700 = 0$$

$$\Rightarrow x(x - 90) + 30(x - 90) = 0$$

$$\Rightarrow (x - 90)(x + 30) = 0$$

We reject  $x = -30$  ( $\because$   $x \neq 0$ )

$$\therefore x = 90$$

$$\Rightarrow BC = 90 \text{ m and}$$

$$AB = (90 + 30) \text{ m} = 120 \text{ m.}$$

Hence, the sides of the rectangular field are 90 m and 120 m.

7. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

**Sol.** Let the larger number be  $x$ .

Then the square of the smaller number is  $8x$ .

According to the given question,

$$\begin{aligned} x^2 - 8x &= 180 \\ \Rightarrow x^2 - 8x - 180 &= 0 \\ \Rightarrow x^2 - 18x + 10x - 180 &= 0 \\ \Rightarrow x(x - 18) + 10(x - 18) &= 0 \\ \Rightarrow (x - 18)(x + 10) &= 0 \\ \Rightarrow x &= 18, -10 \end{aligned}$$

However, the larger number cannot be negative as 8 times of the larger number will be negative and hence, the square of the smaller number will be negative which is not possible.

Therefore, the larger number will be 18 only.

$$\begin{aligned} x &= 18 \\ \therefore y^2 &= 8x = 8 \times 18 = 144 \\ \Rightarrow y &= \pm \sqrt{144} = \pm 12 \\ \therefore \text{Smaller number} &= \pm 12 \end{aligned}$$

Therefore, the numbers are 18 and 12 or 18 and - 12.

- 8.** A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

**Sol.** Let the speed of the train be  $x$  km/h. We are given that 360 km distance is to be travelled at uniform speed of  $x$  km/h.

Time taken to cover the distance

$$= \frac{360}{x} \text{ hours}$$

In case, the speed is increased by 5 km/h, the time required to cover 360 km

$$= \frac{360}{(x+5)} \text{ hours}$$

Now, according to question

$$\begin{aligned} \frac{360}{x} - \frac{360}{x+5} &= 1 \\ \Rightarrow 360 \times \left( \frac{1}{x} - \frac{1}{x+5} \right) &= 1 \\ \Rightarrow \frac{360 \times 5}{x \times (x+5)} &= 1 \\ \Rightarrow x \times (x+5) &= 360 \times 5 \\ \Rightarrow x^2 + 5x &= 1800 \\ \Rightarrow x^2 + 5x - 1800 &= 0 \\ \Rightarrow x^2 - 40x + 45x - 1800 &= 0 \\ \Rightarrow x(x - 40) + 45(x - 40) &= 0 \\ \Rightarrow (x - 40)(x + 45) &= 0 \\ \Rightarrow x &= 40, -45 \\ \Rightarrow x &= 40 \text{ or } -45, \\ &\text{we reject } x = -45 \end{aligned}$$

$\therefore x = 40$   
 $\therefore$  Speed of the train is 40 km/h

- 9.** Two water taps together can fill a tank in  $9\frac{3}{8}$  hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

**Sol.** Let the time taken by the smaller pipe to fill the tank be  $x$  hours.

Time taken by the larger pipe =  $(x - 10)$  hours

Part of tank filled by smaller pipe in 1 hour =  $\frac{1}{x}$

Part of tank filled by larger pipe in 1 hour =  $\frac{1}{x-10}$

It is given that the tank can be filled in

$$9\frac{3}{8} = \frac{75}{8} \text{ hours by both the pipes together.}$$

$$\text{Therefore, } \frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$$

$$\Rightarrow \frac{x-10+x}{x(x-10)} = \frac{8}{75}$$

$$\Rightarrow 75(2x-10) = 8x^2 - 80x$$

$$\Rightarrow 150x - 750 = 8x^2 - 80x$$

$$\Rightarrow 8x^2 - 230x + 750 = 0$$

$$\Rightarrow 8x^2 - 200x - 30x + 750 = 0$$

$$\Rightarrow 8x(x-25) - 30(x-25) = 0$$

$$\Rightarrow (x-25)(8x-30) = 0$$

$$\text{i.e., } x = 25 \text{ or } \frac{30}{8}$$

Time taken by the smaller pipe cannot be  $\frac{30}{8} = 3.75$  hours. As in this case, the time

taken by the larger pipe will be negative, which is logically not possible.

Therefore, time taken individually by the smaller pipe and the larger pipe will be 25 and  $25 - 10 = 15$  hours respectively.

10. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop in intermediate stations). If the average speed of the express train is 11 km/h more than that of the passenger train, find the average speed of the two trains.

**Sol.** Let the average speed of the passenger train be  $x$  km/h.

Then the average speed of express train =  $(x + 11)$  km/h.

Time taken by passenger train for 132 km journey =  $\frac{132}{x}$  hours

Time taken by the express train for 132 km journey =  $\frac{132}{x+11}$

According to given condition.

$$\frac{132}{x} - \frac{132}{x+11} = 1$$

$$\Rightarrow 132 \times \left( \frac{1}{x} - \frac{1}{x+11} \right) = 1$$

$$\Rightarrow 132 \times \frac{11}{x(x+11)} = 1$$

$$\Rightarrow 132 \times 11 = x(x+11)$$

$$\Rightarrow 1452 = x^2 + 11x$$

$$\Rightarrow x^2 + 11x - 1452 = 0$$

$$\Rightarrow a = 1, b = 11 \text{ and } c = -1452$$

$$D = b^2 - 4ac = 121 - 4 \times 1 \times (-1452)$$

$$= 121 + 5808 = 5929$$

$$\Rightarrow \sqrt{D} = \sqrt{5929} = 77$$

$$\text{Then, } x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-11 \pm 77}{2} = -44 \text{ or } 33$$

We reject  $x = -44$

(speed can't be negative)

$$\therefore x = 33$$

Hence, the speed of passenger train = 33 km/h and the speed of express train = 44 km/h

**11.** Sum of the areas of two squares is  $468 \text{ m}^2$ .  
If the difference of their perimeters is 24 m, find the sides of the two squares.

**Sol.** Let the side of one square be  $x \text{ m}$   
Then its perimeter will be  $4x \text{ m}$ .  
Therefore, perimeter of other square will be  $(4x - 24 \text{ m})$   
Then its side will be  $(x - 6) \text{ m}$   
It is given that  $x^2 + (x - 6)^2 = 468$   
 $\Rightarrow x^2 + x^2 - 12x + 36 = 468$

$$\begin{aligned}\Rightarrow 2x^2 - 12x - 432 &= 0 \\ \Rightarrow x^2 - 6x - 216 &= 0 \\ \Rightarrow x^2 + 12x - 18x - 216 &= 0 \\ \Rightarrow x(x + 12) - 18(x + 12) &= 0 \\ \Rightarrow (x + 12)(x - 18) &= 0 \\ \Rightarrow x = -12 \text{ or } x = 18.\end{aligned}$$

However, side of a square cannot be negative.

Hence, the sides of the squares are 18 m and  $(18 - 6) \text{ m} = 12 \text{ m}$