NCERT QUESTIONS WITH SOLUTIONS

EXERCISE : 2.1

1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 - 3x + 7$ (ii) $y^2 + \sqrt{2}$

(iii) $3\sqrt{t} + t\sqrt{2}$ (iv) $y + \frac{2}{y}$

(v) $x^{10} + y^3 + t^{50}$

Sol. (i) $4x^2 - 3x + 7$

This expression is a polynomial in one variable x because there is only one variable (x) in the expression.

(ii) $y^2 + \sqrt{2}$

This expression is a polynomial in one variable y because there is only one variable (y) in the expression.

(iii) $3\sqrt{t} + t\sqrt{2}$

The expression is not a polynomial because in the term $3\sqrt{t}$, the exponent of t is $\frac{1}{2}$, which is not a whole number.

(iv) $y + \frac{2}{y} = y + 2y^{-1}$

The expression is not a polynomial because exponent of y is (–1) in term $\frac{2}{y}$ which in not a whole number.

(v) $x^{10} + y^3 + t^{50}$

The expression is not a polynomial in one variable, it is a polynomial in 3 variables x, y and t.

2. Write the coefficients of x^2 in each of the following :

(i) $2 + x^2 + x$ (ii) $2 - x^2 + x^3$

(iii) $\frac{\pi}{2}x^2 + x$ (iv) $\sqrt{2}x - 1$

Sol. (i) $2 + x^2 + x$

Coefficient of $x^2 = 1$

(ii) $2 - x^2 + x^3$

Coefficient of $x^2 = -1$

(iii) $\frac{\pi}{2}x^2 + x$

Coefficient of $x^2 = \pi/2$

(iv) $\sqrt{2}x - 1$

Coefficient of $x^2 = 0$

3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Sol. One example of a binomial of degree 35 is $3x^{35} - 4$.

One example of monomial of degree 100 is $5x^{100}$.

4. Write the degree of each of the following polynomials :

(i) $5x^3 + 4x^2 + 7x$ (ii) $4 - y^2$

(iii) $5t - \sqrt{7}$ (iv) 3

Sol. (i) $5x^3 + 4x^2 + 7x$

Term with the highest power of x = $5x^3$

Exponent of x in this term = 3

$\therefore$ Degree of this polynomial = 3.

(ii) $4 - y^2$

Term with the highest power of y = $-y^2$

Exponent of y in this term = 2

$\therefore$ Degree of this polynomial = 2

(iii) $5t - \sqrt{7}$

Term with highest power of t = 5t.

Exponent of t in this term = 1

$\therefore$ Degree of this polynomial = 1

(iv) 3

This is a constant which is non-zero

$\therefore$ Degree of this polynomial = 0

5. Classify the following as linear, quadratic and cubic polynomials :

(i) $x^2 + x$ (ii) $x - x^3$

(iii) $y + y^2 + 4$ (iv) $1 + x$

(v) $3t$ (vi) r^2

(vii) $7x^3$

Sol. (i) Quadratic (ii) Cubic

(iii) Quadratic (iv) Linear

(v) Linear (vi) Quadratic

(vii) Cubic

EXERCISE : 2.2

1. Find the value of the polynomial $5x - 4x^2 + 3$ at

(i) $x = 0$ (ii) $x = -1$

(iii) $x = 2$

Sol. Let $f(x) = 5x - 4x^2 + 3$

(i) Value of f(x) at x = 0 = f(0)

$= 5(0) - 4(0)^2 + 3 = 3$

(ii) Value of f(x) at x = –1 = f(–1)

$= 5(-1) - 4(-1)^2 + 3 = -5 - 4 + 3 = -6$

(iii) Value of f(x) at x = 2 = f(2)

$= 5(2) - 4(2)^2 + 3$

$= 10 - 16 + 3 = -3$

2. Find p(0), p(1) and p(2) for each of the following polynomials :

(i) $p(y) = y^2 - y + 1$

(ii) $p(t) = 2 + t + 2t^2 - t^3$

(iii) $p(x) = x^3$

(iv) $p(x) = (x - 1)(x + 1)$

Sol. (i) $p(y) = y^2 - y + 1$

$\therefore$ $p(0) = (0)^2 - (0) + 1 = 1$,

$p(1) = (1)^2 - (1) + 1 = 1$,

$p(2) = (2)^2 - (2) + 1 = 4 - 2 + 1 = 3$.

(ii) $p(t) = 2 + t + 2t^2 - t^3$

$p(0) = 2 + 0 + 2(0)^2 - (0)^3 = 2$

$p(1) = 2 + 1 + 2(1)^2 - (1)^3 = 2 + 1 + 2 - 1 = 4$

$p(2) = 2 + 2 + 2(2)^2 - (2)^3 = 2 + 2 + 8 - 8 = 4$

(iii) $p(x) = x^3$

$p(0) = (0)^3 = 0$

$p(1) = (1)^3 = 1$

$p(2) = (2)^3 = 8$

(iv) $p(x) = (x - 1)(x + 1)$

$p(0) = (0 - 1)(0 + 1) = (-1)(1) = -1$

$p(1) = (1 - 1)(1 + 1) = 0(2) = 0$

$p(2) = (2 - 1)(2 + 1) = (1)(3) = 3$

3. Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x) = 3x + 1,\ x = -\frac{1}{3}$

(ii) $p(x) = 5x - \pi,\ x = \frac{4}{5}$

(iii) $p(x) = x^2 - 1,\ x = 1, -1$

(iv) $p(x) = (x + 1)(x - 2),\ x = -1, 2$

(v) $p(x) = x^2,\ x = 0$

(vi) $p(x) = \ell x + m,\ x = -\frac{m}{\ell}$

(vii) $p(x) = 3x^2 - 1,\ x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

(viii) $p(x) = 2x + 1,\ x = \frac{1}{2}$

Sol. (i) $p(x) = 3x + 1,\ x = -\frac{1}{3}$

$$p\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right) + 1 = -1 + 1 = 0$$

$-\frac{1}{3}$ is a zero of p(x).

(ii) $p(x) = 5x - \pi,\ x = \frac{4}{5}$

$$p\left(\frac{4}{5}\right) = 5\left(\frac{4}{5}\right) - \pi = 4 - \pi \neq 0$$

$\therefore$ $\frac{4}{5}$ is not a zero of p(x)

(iii) $p(x) = x^2 - 1,\ x = 1, -1$

$p(1) = (1)^2 - 1 = 1 - 1 = 0$

$p(-1) = (-1)^2 - 1 = 1 - 1 = 0$

$\therefore$ 1, –1 are zeroes of p(x)

(iv) $p(x) = (x + 1)(x - 2),\ x = -1, 2$

$p(-1) = (-1 + 1)(-1 - 2) = (0)(-3) = 0$

$p(2) = (2 + 1)(2 - 2) = (3)(0) = 0$

$\therefore$ –1, 2 are zeroes of p(x)

(v) $p(x) = x^2,\ x = 0$

$p(0) = 0$

$\therefore$ 0 is a zero of p(x)

(vi) $p(x) = \ell x = m,\ x = \frac{-m}{\ell}$

$$p\left(\frac{-m}{\ell}\right) = \ell\left(\frac{-m}{\ell}\right) + m = -m + m = 0$$

$\therefore$ $\frac{-m}{\ell}$ is a zero of p(x).

(vii) $p(x) = 3x^2 - 1,\ x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

$$p\left(-\frac{1}{\sqrt{3}}\right) = 3\left(-\frac{1}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{1}{3}\right) - 1$$

$= 1 - 1 = 0$

$$p\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{4}{3}\right) - 1$$

$= 4 - 1 = 3 \neq 0$

So, $-\frac{1}{\sqrt{3}}$ is a zero of p(x) and $\frac{2}{\sqrt{3}}$ is not a zero of p(x).

(viii) $p(x) = 2x + 1,\ x = \frac{1}{2}$

$$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1 = 1 + 1 = 2 \neq 0$$

$\therefore$ $\frac{1}{2}$ is not a zero of p(x).

4. Find the zero of the polynomial in each of the following cases :

(i) $p(x) = x + 5$ (ii) $p(x) = x - 5$

(iii) $p(x) = 2x + 5$ (iv) $p(x) = 3x - 2$

(v) $p(x) = 3x$ (vi) $p(x) = ax,\ a \neq 0$

(vii) $p(x) = cx + d,\ c \neq 0$, c, d are real numbers.

Sol. (i) $p(x) = x + 5$

$p(x) = 0$

$\Rightarrow$ $x + 5 = 0 \Rightarrow x = -5$

$\therefore$ –5 is zero of the polynomial p(x).

(ii) $p(x) = x - 5$

$p(x) = 0$

$x - 5 = 0$

or $x = 5$

$\therefore$ 5 is zero of polynomial p(x).

(iii) $p(x) = 2x + 5$

$p(x) = 0$

$2x + 5 = 0$

$2x = -5 \Rightarrow x = -\frac{5}{2}$

$\therefore$ $-\frac{5}{2}$ is zero of polynomial p(x).

(iv) $p(x) = 3x - 2$

$p(x) = 0 \Rightarrow 3x - 2 = 0$

or $x = \frac{2}{3}$

$\therefore$ $\frac{2}{3}$ is zero of polynomial p(x).

(v) $p(x) = 3x$

$p(x) = 0 \Rightarrow 3x = 0$

or $x = 0$

$\therefore$ 0 is zero of polynomial p(x).

(vi) $p(x) = ax, a \neq 0$

$\Rightarrow$ $ax = 0$ or $x = 0$

$\therefore$ 0 is zero of p(x)

(vii) $p(x) = cx + d, c \neq 0$, c, d are real numbers

$cx + d = 0 \Rightarrow cx = -d$

$x = \frac{-d}{c}$

$\therefore$ $\frac{-d}{c}$ is zero of polynomial p(x).

EXERCISE : 2.3

1. Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by :

(i) $x + 1$ (ii) $x - \frac{1}{2}$

(iii) x (iv) $x + \pi$

(v) $5 + 2x$

Sol. (i) $x + 1$

$x + 1 = 0$

$\Rightarrow x = -1$

$\therefore$ Remainder $= p(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1 = -1 + 3 - 3 + 1 = 0$

(ii) $x - \frac{1}{2}$

$x - \frac{1}{2} = 0 \Rightarrow x = \frac{1}{2}$

$\therefore$ Remainder $= p\left(\frac{1}{2}\right)$

$= \left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 1 = \frac{1}{8} + \frac{3}{4} + \frac{3}{2} + 1$

$= \frac{27}{8}$

(iii) x

$x = 0$

Remainder = p(0)

$= (0)^3 + 3(0)^2 + 3(0) + 1 = 1$

(iv) $x + \pi$

$x + \pi = 0 \Rightarrow x = -\pi$

$\therefore$ Remainder $= p(-\pi)$

$= (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1$

$= -\pi^3 + 3\pi^2 - 3\pi + 1$

(v) $5 + 2x$

$5 + 2x = 0 \Rightarrow x = -5/2$

$\therefore$ Remainder $= p(-5/2)$

$= \left(\frac{-5}{2}\right)^3 + 3\left(\frac{-5}{2}\right)^2 + 3\left(\frac{-5}{2}\right) + 1$

$= \frac{-125}{8} + \frac{75}{4} - \frac{15}{2} + 1 = -\frac{27}{8}$

2. Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

Sol. Let $p(x) = x^3 - ax^2 + 6x - a$

$x - a = 0 \Rightarrow x = a$

$\therefore$ Remainder $= (a)^3 - a(a)^2 + 6(a) - a$

$= a^3 - a^3 + 6a - a = 5a$

3. Check whether $7 + 3x$ is a factor of $3x^3 + 7x$.

Sol. $7 + 3x$ will be a factor of $3x^3 + 7x$ only if $7 + 3x$ divides $3x^3 + 7x$ leaving 0 as remainder.

Let $p(x) = 3x^3 + 7x$

$7 + 3x = 0 \Rightarrow 3x = -7 \Rightarrow x = -7/3$

$\therefore$ Remainder

$3\left(-\frac{7}{3}\right)^3 + 7\left(-\frac{7}{3}\right) = \frac{-343}{9} - \frac{49}{3} = \frac{-490}{9} \neq 0$

so, $7 + 3x$ is not a factor of $3x^3 + 7x$.

EXERCISE : 2.4

1. Determine which of the following polynomials has $(x + 1)$ a factor :

(i) $x^3 + x^2 + x + 1$

(ii) $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

(iv) $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

Sol. (i) $x^3 + x^2 + x + 1$

Let $p(x) = x^3 + x^2 + x + 1$

The zero of $x + 1$ is -1

$p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 + 1 = 0$

By Factor theorem $x + 1$ is a factor of $p(x)$.

(ii) $x^4 + x^3 + x^2 + x + 1$

Let $p(x) = x^4 + x^3 + x^2 + x + 1$

The zero of $x + 1$ is -1

$p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1 = 1 \neq 0$

By Factor theorem $x + 1$ is not a factor of $p(x)$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

Let $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

Zero of $x + 1$ is -1

$p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$

$= 1 - 3 + 3 - 1 + 1 = 1 \neq 0$

By Factor theorem $x + 1$ is not a factor of $p(x)$

(iv) Let $p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

zero of $x + 1$ is -1

$p(-1) = (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2}$

$= -1 - 1 + 2 + \sqrt{2} + \sqrt{2} = 2\sqrt{2} \neq 0$

By Factor theorem, $x + 1$ is not a factor of $p(x)$.

2. Use the factor theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases :

(i) $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$.

(ii) $p(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x + 2$.

(iii) $p(x) = x^3 - 4x^2 + x + 6$; $g(x) = x - 3$

Sol. (i) $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$.

$g(x) = 0 \Rightarrow x + 1 = 0 \Rightarrow x = -1$

$\therefore$ Zero of $g(x)$ is -1

Now, $p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$

$= -2 + 1 + 2 - 1 = 0$

$\therefore$ By Factor theorem, $g(x)$ is a factor of $p(x)$.

(ii) Let $p(x) = x^3 + 3x^2 + 3x + 1$,

$g(x) = x + 2$

$g(x) = 0 \Rightarrow x + 2 = 0 \Rightarrow x = -2$

$\therefore$ Zero of $g(x)$ is -2

Now, $p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$

$= -8 + 12 - 6 + 1 = -1$

$\therefore$ By Factor theorem, $g(x)$ is not a factor of $p(x)$

(iii) $p(x) = x^3 - 4x^2 + x + 6$, $g(x) = x - 3$

$g(x) = 0$

$\Rightarrow x - 3 = 0 \Rightarrow x = 3$

$\therefore$ Zero of $g(x) = 3$

Now $p(3) = 3^3 - 4(3)^2 + 3 + 6$

$= 27 - 36 + 3 + 6 = 0$

$\therefore$ By Factor theorem, $g(x)$ is a factor of $p(x)$.

3. Find the value of k, if $x - 1$ is a factor of $p(x)$ in each of the following cases :

(i) $p(x) = x^2 + x + k$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

(iv) $p(x) = kx^2 - 3x + k$

Sol. (i) $p(x) = x^2 + x + k$

If $x - 1$ is a factor of $p(x)$, then $p(1) = 0$

$\Rightarrow (1)^2 + (1) + k = 0$

$\Rightarrow 1 + 1 + k = 0$

$\Rightarrow 2 + k = 0$

$\Rightarrow k = -2$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

If $(x - 1)$ is a factor of $p(x)$ then $p(1) = 0$

$\Rightarrow$ $2(1)^2 + k(1) + \sqrt{2} = 0$

$\Rightarrow$ $2 + k + \sqrt{2} = 0$

$\Rightarrow$ $k = -(2 + \sqrt{2})$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

If $(x - 1)$ is a factor of $p(x)$ then $p(1) = 0$

$k(1)^2 - \sqrt{2}(1) + 1 = 0$

$\Rightarrow$ $k - \sqrt{2} + 1 = 0$

$k = \sqrt{2} - 1$

(iv) $p(x) = kx^2 - 3x + k$

If $(x-1)$ is a factor of $p(x)$ then $p(1) = 0$

$\Rightarrow$ $k(1)^2 - 3(1) + k = 0$

$2k = 3$

$k = 3/2$

4. Factorise :

(i) $12x^2 - 7x + 1$ (ii) $2x^2 + 7x + 3$

(iii) $6x^2 + 5x - 6$ (iv) $3x^2 - x - 4$

Sol. (i) $12x^2 - 7x + 1$

$= 12x^2 - 4x - 3x + 1$

$= 4x(3x - 1) - 1(3x - 1)$

$= (3x - 1)(4x - 1)$

(ii) $2x^2 + 7x + 3$

$= 2x^2 + 6x + x + 3$

$= 2x(x + 3) + 1(x + 3)$

$= (x + 3)(2x + 1)$

(iii) $6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$

$= 3x(2x + 3) - 2(2x + 3)$

$= (3x - 2)(2x + 3)$

(iv) $3x^2 - x - 4 = 3x^2 - 4x + 3x - 4$

$= x(3x - 4) + 1(3x - 4)$

$= (x + 1)(3x - 4)$

5. Factorise :

(i) $x^3 - 2x^2 - x + 2$

(ii) $x^3 - 3x^2 - 9x - 5$

(iii) $x^3 + 13x^2 + 32x + 20$

(iv) $2y^3 + y^2 - 2y - 1$

Sol. (i) $x^3 - 2x^2 - x + 2$

Let $p(x) = x^3 - 2x^2 - x + 2$

By trial, we find that

$p(1) = (1)^3 - 2(1)^2 - (1) + 2$

$= 1 - 2 - 1 + 2 = 0$

$\therefore$ By Factor Theorem, $(x - 1)$ is a factor of $p(x)$.

Now, $x^3 - 2x^2 - x + 2$

$= x^2(x - 1) - x(x - 1) - 2(x - 1)$

$= (x - 1)(x^2 - x - 2)$

$= (x - 1)(x^2 - 2x + x - 2)$

$= (x - 1)\{x(x - 2) + 1(x - 2)\}$

$= (x - 1)(x - 2)(x + 1)$

(ii) $x^3 - 3x^2 - 9x - 5$

Let $p(x) = x^3 - 3x^2 - 9x - 5$

By trial, we find that

$p(-1) = (-1)^3 - 3(-1)^2 - 9(-1) - 5$

$= -1 - 3 + 9 - 5 = 0$

$\therefore$ By Factor Theorem, $x = -1$ or $x + 1$ is factor of $p(x)$.

Now, $x^3 - 3x^2 - 9x - 5$

$= x^2(x + 1) - 4x(x + 1) - 5(x + 1)$

$= (x + 1)(x^2 - 4x - 5)$

$= (x + 1)(x^2 - 5x + x - 5)$

$= (x + 1)\{x(x - 5) + 1(x - 5)\}$

$= (x + 1)^2(x - 5)$

(iii) $x^3 + 13x^2 + 32x + 20$

Let $p(x) = x^3 + 13x^2 + 32x + 20$

By trial, we find that

$p(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20$

$= -1 + 13 - 32 + 20 = 0$

$\therefore$ By Factor Theorem, $x = -1$ or $x + 1$ is a factor of $p(x)$

$x^3 + 13x^2 + 32x + 20$

$= x^2(x + 1) + 12(x)(x + 1) + 20(x + 1)$

$= (x + 1)(x^2 + 12x + 20)$

$= (x + 1)(x^2 + 2x + 10x + 20)$

$= (x + 1)\{x(x + 2) + 10(x + 2)\}$

$= (x + 1)(x + 2)(x + 10)$

(iv) $2y^3 + y^2 - 2y - 1$

$p(y) = 2y^3 + y^2 - 2y - 1$

By trial, we find that

$p(1) = 2(1)^3 + (1)^2 - 2(1) - 1 = 0$

$\therefore$ By Factor Theorem, $(y - 1)$ is a factor of

$p(y) = 2y^3 + y^2 - 2y - 1$

$= 2y^2(y - 1) + 3y(y - 1) + 1(y - 1)$

$= (y - 1)(2y^2 + 3y + 1)$

$= (y - 1)(2y^2 + 2y + y + 1)$

$= (y - 1)\{2y(y + 1) + 1(y + 1)\}$

$= (y - 1)(2y + 1)(y + 1)$

EXERCISE : 2.5

1. Use suitable identities to find the following products :

(i) $(x + 4)(x + 10)$

(ii) $(x + 8)(x - 10)$

(iii) $(3x + 4)(3x - 5)$

(iv) $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$

(v) $(3 - 2x)(3 + 2x)$

Sol. (i) $(x + 4)(x + 10)$

$= x^2 + (4 + 10)x + (4)(10)$

$= x^2 + 14x + 40$

(ii) $(x + 8)(x - 10)$

$= (x + 8)\{x + (-10)\}$

$= x^2 + \{8 + (-10)\}x + 8(-10)$

$= x^2 - 2x - 80$

(iii) $(3x + 4)(3x - 5)$

$= (3x + 4)(3x - 5) = (3x + 4)(3x + (-5))$

$= (3x)^2 + \{4 + (-5)\}(3x) + 4(-5)$

$= 9x^2 - 3x - 20$

(iv) $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$

Let, $y^2 = x$

$\Rightarrow \left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right) = \left(x + \frac{3}{2}\right)\left(x - \frac{3}{2}\right)$

$= x^2 - \frac{9}{4}$

[using identity $(a + b)(a - b) = a^2 - b^2$]

Substituting $x = y^2$, we get

$= (y^2)^2 - \frac{9}{4}$

$= y^4 - \frac{9}{4}$

(v) $(3 - 2x)(3 + 2x)$

$(3)^2 - (2x)^2 = 9 - 4x^2$

[using identity $(a + b)(a - b) = a^2 - b^2$]

2. Evaluate the following products without multiplying directly:

(i) 103×107 (ii) 95×96

(iii) 104×96

Sol. (i) $103 \times 107 = (100 + 3) \times (100 + 7)$

$= (100)^2 + (3 + 7)(100) + (3)(7)$

$= 10000 + 1000 + 21 = 11021$

Alternate solution:

$103 \times 107 = (105 - 2) \times (105 + 2)$

$= (105)^2 - (2)^2 = (100 + 5)^2 - 4$

$= (100)^2 + 2(100)(5) + (5)^2 - 4$

$= 10000 + 1000 + 25 - 4$

$= 11021.$

(ii) 95×96

$= (90 + 5) \times (90 + 6)$

$= (90)^2 + (5 + 6)\,90 + (5)(6)$

$= 8100 + 990 + 30 = 9120$

(iii) 104×96

$= (100 + 4) \times (100 - 4)$

[using identity $(a + b)(a - b) = a^2 - b^2$]

$= (100)^2 - (4)^2 = 10000 - 16$

$= 9984$

3. Factorise the following using appropriate identities :

(i) $9x^2 + 6xy + y^2$

(ii) $4y^2 - 4y + 1$

(iii) $x^2 - \frac{y^2}{100}$

Sol. (i) $9x^2 + 6xy + y^2$

$= (3x)^2 + 2(3x)(y) + (y)^2$

$= (3x + y)^2$

$= (3x + y)(3x + y)$

(ii) $4y^2 - 4y + 1$

$= (2y)^2 - 2(2y)(1) + (1)^2$

$= (2y - 1)^2 = (2y - 1)(2y - 1)$

(iii) $x^2 - \frac{y^2}{100}$

[using identity $(a + b)(a - b) = a^2 - b^2$]

$x^2 - \left(\frac{y}{10}\right)^2 = \left(x + \frac{y}{10}\right)\left(x - \frac{y}{10}\right)$

4. Expand each of the following using suitable identities :

(i) $(x + 2y + 4z)^2$ (ii) $(2x - y + z)^2$

(iii) $(-2x + 3y + 2z)^2$ (iv) $(3a - 7b - c)^2$

(v) $(-2x + 5y - 3z)^2$

(vi) $\left(\frac{1}{4}a - \frac{1}{2}b + 1\right)^2$

Sol. (i) $(x + 2y + 4z)^2 = (x)^2 + (2y)^2 + (4z)^2 + 2(x)(2y) + 2(2y)(4z) + 2(4z)(x)$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8zx$

(ii) $(2x - y + z)^2$

$= (2x - y + z)(2x - y + z)$

$= (2x)^2 + (-y)^2 + (z)^2 + 2(2x)(-y) + 2(-y)(z) + 2(z)(2x)$

$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4zx$

(iii) $(-2x + 3y + 2z)^2$

$= (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(-2x)(2z) + 2(3y)(2z)$

$= 4x^2 + 9y^2 + 4z^2 - 12xy - 8xz + 12yz$

(iv) $(3a - 7b - c)^2 = (3a - 7b - c)(3a - 7b - c)$

$= (3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b) + 2(3a)(-c) + 2(-7b)(-c)$

$= 9a^2 + 49b^2 + c^2 - 42ab - 6ac + 14bc$

(v) $(-2x + 5y - 3z)^2$

$= (-2x + 5y - 3z)(-2x + 5y - 3z)$

$= (-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(5y) + 2(-2x)(-3z) + 2(-3z)(5y)$

$= 4x^2 + 25y^2 + 9z^2 - 20xy + 12xz - 30yz$

(vi) $\left(\frac{1}{4}a - \frac{1}{2}b + 1\right)^2$

$= \left(\frac{1}{4}a - \frac{1}{2}b + 1\right)\left(\frac{1}{4}a - \frac{1}{2}b + 1\right)$

$= \left(\frac{1}{4}a\right)^2 + \left(-\frac{1}{2}b\right)^2 + (1)^2 + 2\left(\frac{1}{4}a\right)\left(-\frac{1}{2}b\right)$

$+ 2\left(\frac{1}{4}a\right)(1)^2 + 2\left(-\frac{1}{2}b\right)(1)$

$= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a$

5. Factorise :

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

Sol. (i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(2x)$

$= \{2x + 3y + (-4z)\}^2 = (2x + 3y - 4z)^2$

$= (2x + 3y - 4z)(2x + 3y - 4z)$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

$= \left(-\sqrt{2}x\right)^2 + y^2 + \left(2\sqrt{2}z\right)^2 + 2\left(-\sqrt{2}x\right)y + 2y\left(2\sqrt{2}z\right) + 2\left(2\sqrt{2}z\right)\left(-\sqrt{2}x\right)$

$= \left(-\sqrt{2}x + y + 2\sqrt{2}z\right)^2$

6. Write the following cubes in expanded form :

(i) $(2x + 1)^3$ (ii) $(2a - 3b)^3$

(iii) $\left(\frac{3}{2}x+1\right)^3$ (iv) $\left(x-\frac{2}{3}y\right)^3$

Sol. (i) $(2x + 1)^3 = (2x)^3 + (1)^3 + 3(2x)(1)(2x + 1)$

$= 8x^3 + 1 + 6x(2x + 1)$

$= 8x^3 + 1 + 12x^2 + 6x$

$= 8x^3 + 12x^2 + 6x + 1$

(ii) $(2a - 3b)^3 = (2a)^3 - (3b)^3 - 3(2a)(3b)(2a-3b)$

$= 8a^3 - 27b^3 - 18ab(2a - 3b)$

$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$

(iii) $\left(\frac{3}{2}x+1\right)^3 = \left(\frac{3}{2}x\right)^3 + (1)^3 + 3\left(\frac{3}{2}x\right)(1)\left(\frac{3}{2}x+1\right)$

$= \frac{27}{8}x^3 + 1 + \frac{27}{4}x^2 + \frac{9}{2}x$

$= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$

(iv) $\left(x-\frac{2}{3}y\right)^3 = x^3 - \left(\frac{2}{3}y\right)^3 - 3x\left(\frac{2}{3}y\right)\left(x-\frac{2}{3}y\right)$

$= x^3 - \frac{8}{27}y^3 - 2xy\left(x-\frac{2}{3}y\right)$

$= x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$

7. Evaluate the following using suitable identities :

(i) $(99)^3$ (ii) $(102)^3$

(iii) $(998)^3$

Sol. (i) $(99)^3 = (100 - 1)^3$

$= (100)^3 - (1)^3 - 3(100)(1)(100 - 1)$

$= 1000000 - 1 - 300(100 - 1)$

$= 1000000 - 1 - 30000 + 300$

$= 970299$

(ii) $(102)^3 = (100 + 2)^3$

$= (100)^3 + (2)^3 + 3(100)(2)(100 + 2)$

$= 1000000 + 8 + 600(100 + 2)$

$= 1000000 + 8 + 60000 + 1200$

$= 1061208$

(iii) $(998)^3 = (1000-2)^3$

$= (1000)^3 - (2)^3 - 3(1000)(2)(1000-2)$

$= 1000000000 - 8 - 6000(1000-2)$

$= 994011992$

8. Factorise each of the following :

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii) $27 - 125a^3 - 135a + 225a^2$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Sol. (i) $8a^3 + b^3 + 12a^2b + 6ab^2$

$= (2a)^3 + (b)^3 + 3(2a)(b)(2a + b)$

$= (2a + b)^3 = (2a + b)(2a + b)(2a + b)$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

$= (2a)^3 + (-b)^3 + 3(2a)^2(-b) + 3(2a)(-b)^2$

$= (2a - b)^3$

(iii) $27 - 125a^3 - 135a + 225a^2$

$= 3^3 - (5a)^3 - 3(3)(5a)(3-5a)$

$= (3 - 5a)^3$

(iv) $64a^3 - 27b^3 - 144a^2b + 180ab^2$

$= (4a)^3 - (3b)^3 - 3(4a)(3b)(4a - 3b)$

$= (4a - 3b)^3$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4p}$

$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3(3p)\left(\frac{1}{6}\right)\left(3p-\frac{1}{6}\right)$

$= \left(3p-\frac{1}{6}\right)^3 = \left(3p-\frac{1}{6}\right)\left(3p-\frac{1}{6}\right)\left(3p-\frac{1}{6}\right)$

9. Verify :

(i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

(ii) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Sol. (i) $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$\Rightarrow x^3 + y^3 = (x + y)^3 - 3xy(x + y)$

$\Rightarrow x^3 + y^3 = (x + y)\{(x + y)^2 - 3xy\}$

$\Rightarrow x^3 + y^3 = (x + y)(x^2 + 2xy + y^2 - 3xy)$

$\Rightarrow x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

(ii) $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$\Rightarrow x^3 - y^3 = (x - y)^3 + 3xy(x - y)$

$\Rightarrow x^3 - y^3 = (x - y)[(x - y)^2 + 3xy]$

$\Rightarrow x^3 - y^3 = (x - y)(x^2 + y^2 - 2xy + 3xy)$

$\Rightarrow x^3 - y^3 = (x - y)(x^2 + y^2 + xy)$

10. Factorise each of the following :

(i) $27y^3 + 125z^3$

(ii) $64m^3 - 343n^3$

Sol. (i) $27y^3 + 125z^3 = (3y)^3 + (5z)^3$

$= (3y + 5z)\{(3y)^2 - (3y)(5z) + (5z)^2\}$

$= (3y + 5z)(9y^2 - 15yz + 25z^2)$

(ii) $64m^3 - 343n^3$

$= (4m)^3 - (7n)^3$

$= (4m - 7n)\{16m^2 + 4m.7n + (7n)^2\}$

$= (4m - 7n)(16m^2 + 28mn + 49n^2)$

11. Factorise : $27x^3 + y^3 + z^3 - 9xyz$

Sol. $27x^3 + y^3 + z^3 - 9xyz$

$= (3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$

$= (3x + y + z)((3x)^2 + (y)^2 + (z)^2 - (3x) - (y)(z) - (z)(3x))$

$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3zx)$

12. Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

Sol. $\frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

$= \frac{1}{2}(x + y + z)[(x^2 - 2xy + y^2) + (y^2 - 2yz + z^2) + (z^2 - 2zx + x^2)]$

$= \frac{1}{2}(x + y + z)(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx)$

$= \frac{1}{2}(x + y + z)\,2(x^2 + y^2 + z^2 - xy - yz - zx)$

$= \frac{1}{2}(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= x^3 + y^3 + z^3 - 3xyz$

13. If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$

Sol. We know

$x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

Given : $x + y + z = 0$

$= (0)(x^2 + y^2 + z^2 - xy - yz - zx) = 0$

or $x^3 + y^3 + z^3 = 3xyz$

14. Without actually calculating the cubes, find the value of each of the following :

(i) $(-12)^3 + (7)^3 + (5)^3$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Sol. (i) $(-12)^3 + (7)^3 + (5)^3$

$-12 + 7 + 5 = 0$

$(-12)^3 + (7)^3 + (5)^3$

$= 3(-12)(7)(5) = -1260$

[using identity]

if $a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

$28 - 15 - 13 = 0$

$(28)^3 + (-15)^3 + (-13)^3$

$= 3(28)(-15)(-13) = 16380$

[using identity]

if $a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$

15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given :

(i) Area = $25a^2 - 35a + 12$

(ii) $\text{Area} = 35y^2 + 13y - 12$

Sol. (i) Area = $25a^2 - 35a + 12$

$= 25a^2 - 20a - 15a + 12$

$= 5a(5a - 4) - 3(5a - 4)$

$= (5a - 3)(5a - 4)$

Here, Length = $5a - 3$, Breadth = $5a - 4$

(ii) $35y^2 + 13y - 12$

$= 35y^2 + 28y - 15y - 12$

$= 7y(5y + 4) - 3(5y + 4)$

$= (5y + 4)(7y - 3)$

Here, Length = $5y + 4$, Breadth = $7y - 3$.

16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) Volume : $3x^2 - 12x$

(ii) Volume : $12ky^2 + 8xy - 20k$

Sol. (i) Volume = $3x^2 - 12x$

$= 3x(x - 4) = 3 \times x \times (x - 4)$

$\therefore$ Dimensions are 3 units, x-units and $(x - 4)$ units

(ii) $12ky^2 + 8ky - 20k$

$= 4k(3y^2 + 2y - 5) = 4k(3y^2 + 5y - 3y - 5)$

$= 4k\{y(3y + 5) - 1(3y + 5)\}$

$= 4k(3y + 5)(y - 1)$

$\therefore$ Dimensions of cuboid are $4k$, $3y + 5$, $y - 1$