NCERT QUESTIONS WITH SOLUTIONS

EXERCISE : 12.1

Unless stated otherwise, take $\pi = \frac{22}{7}$

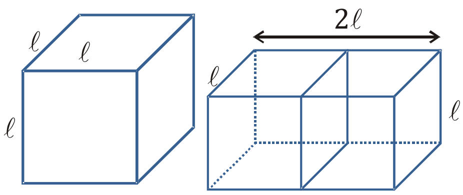
1. 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

Sol. Let ℓ cm be the length of an edge of the cube having volume = 64 cm^3.

Then, $\ell^3 = 64 = (4)^3 \Rightarrow \ell = 4$ cm

Now, the dimensions of the resulting cuboid made by joining two cubes (see figure) are

8 cm × 4 cm × 4 cm (i.e., length = 8 cm, breadth = 4 cm and height = 4 cm)



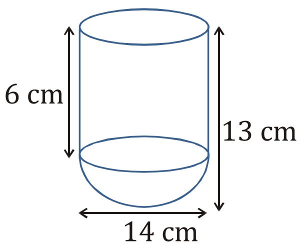
Surface area of cuboid = $2(\ell b + bh + h\ell)$

$= 2(8 \times 4 + 4 \times 4 + 4 \times 8)$

$= 2(32 + 16 + 32) = 2 \times 80 = 160\ cm^2$

2. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

Sol. For hemispherical part,



radius (r) = $\frac{14}{2}$ = 7 cm

∴ Curved surface area = $2\pi r^2$

$= 2 \times \frac{22}{7} \times 7 \times 7\ cm^2 = 308\ cm^2$

Total height of vessel = 13 cm

∴ Height of cylinder = (13 – 7) cm = 6 cm and radius (r) = 7 cm

∴ Curved surface area of cylinder = $2\pi rh$

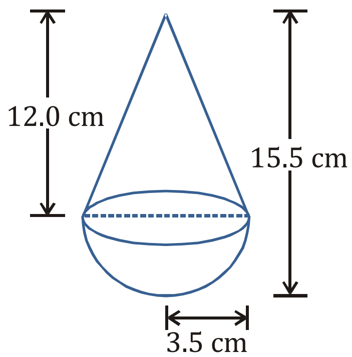
$= 2 \times \frac{22}{7} \times 7 \times 6\ cm^2 = 264\ cm^2$

∴ Inner surface area of vessel = Curved surface area of hemispherical part + Curved surface area of cylinder

$= (308 + 264)\ cm^2 = 572\ cm^2$

3. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. find the total surface area of the toy.

Sol. Let r and h be the radius of cone, hemisphere and height of cone



∴ h = (15.5 – 3.5) cm = 12.0 cm

Also $\ell^2 = h^2 + r^2$

$= 12^2 + (3.5)^2$

$= 156.25$

∴ $\ell = 12.5$ cm

Curved surface area of the conical part = $\pi r\ell$

Curved surface area of the hemispherical part = $2\pi r^2$

Total surface area of the toy = $\pi r\ell + 2\pi r^2$

$= \pi r(\ell + 2r)$

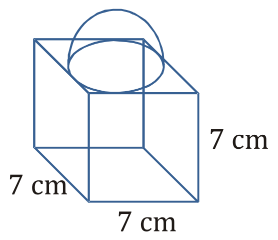
$= \frac{22}{7} \times \frac{35}{10}$ (12.5 + 2 × 3.5) cm^2

= 11 × (12.5 + 7) cm^2 = 11 × 19.5 cm^2

= 214.5 cm^2

4. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

Sol.



On 7 cm × 7 cm base of the cubical block, we can mount hemisphere having greatest diameter equal to 7 cm.

Here, the radius of the hemisphere = 3.5 cm.

Now, the surface area of the solid made in below figure.

The total surface area of the cube + The curved surface area of the hemisphere – The area of the base of the hemisphere.

= {6 × (7)^2 + 2π × (3.5)^2 – π × (3.5)^2} cm^2

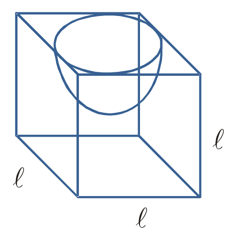
($\because$ the part of the top of the cubical part which is covered by the hemisphere is not visible outside)

$= \left\{6 \times 49 + \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10}\right\}$ cm^2

$= \left\{294 + 11 \times \frac{35}{10}\right\}$ cm^2 = 332.5 cm^2

5. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter ℓ of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Sol. Let ℓ be the side of the cube.



$\therefore$ The greatest diameter of the hemisphere = ℓ

$\Rightarrow$ Radius of the hemisphere = $\frac{\ell}{2}$

$\therefore$ Surface area of hemisphere = $2\pi r^2$

$= 2 \times \pi \times \frac{\ell}{2} \times \frac{\ell}{2} = \frac{\pi\ell^2}{2}$

Base area of the hemisphere

$= \pi\left(\frac{\ell}{2}\right)^2 = \frac{\pi\ell^2}{4}$

Surface area of the cube = $6 \times \ell^2 = 6\ell^2$

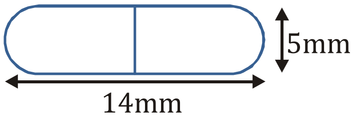
$\therefore$ Surface area of the remaining solid

$= 6\ell^2 + \frac{\pi\ell^2}{2} - \frac{\pi\ell^2}{4}$

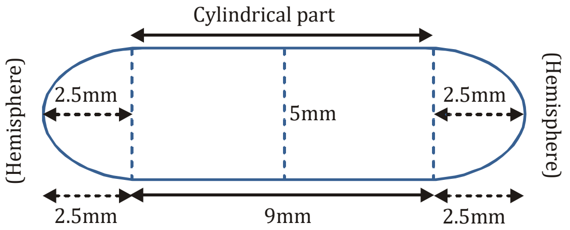
$= \frac{24\ell^2 + 2\pi\ell^2 - \pi\ell^2}{4} = \frac{24\ell^2 + \pi\ell^2}{4}$

$= \frac{\ell^2}{4}$ (24 + π) sq. units.

6. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see figure). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.



Sol. Surface area of the cylindrical part = $2\pi \times r \times h$



$= 2\pi \times \left(\frac{5}{2}\right) \times 9 \text{ mm}^2 = 45\pi \text{ mm}^2$

Sum of the curved surface areas of two hemispherical parts.

$= 2\left\{2\pi \times \left(\frac{5}{2}\right)^2\right\} \text{mm}^2 = 25\pi \text{ mm}^2$

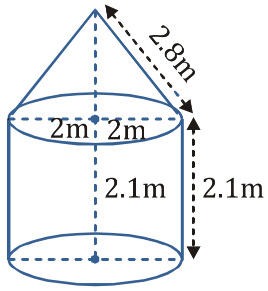
Total surface area of the capsule

$= 45\pi + 25\pi \text{ mm}^2 = 70\pi \text{ mm}^2$

$= 70 \times \frac{22}{7} \text{ mm}^2 = 220 \text{ mm}^2$

7. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2·1 m and 4 m respectively, and the slant height of the top is 2·8 m, find the area of the canvas used for making the tent. Also find the cost of the canvas of the tent at the rate of ₹ 500 per m² (Note that the base of the tent will not be covered with canvas).

Sol.



Radius of the cylindrical base = 2m and height = 2.1 m. The curved surface area of the cylindrical part

$= 2\pi \times (2) \times (2.1) \text{ m}^2$ (i.e., $2\pi rh$)

$= 4 \times \frac{22}{7} \times 2.1 \text{ m}^2$

$= 26.4 \text{ m}^2$

Now, for the conical part,

we have r = 2m and

ℓ (slant height) = 2.8 m

The curved surface area of the conical part = $\pi r \ell$

$= \frac{22}{7} \times 2 \times 2.8 \text{ m}^2$

$= 17.6 \text{ m}^2$

Then the area of the canvas

$= 26.4 \text{ m}^2 + 17.6 \text{ m}^2 = 44 \text{ m}^2$

Total cost of the canvas at the rate of ₹ 500 per m²

= ₹ 500 × 44 = ₹ 22000

8. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm².

Sol. For cylinder part

Height = 2.4 cm and diameter = 1.4 cm

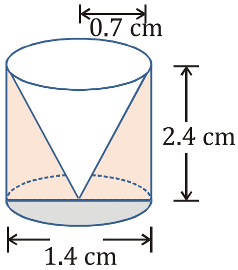
$\Rightarrow$ Radius (r) = 0.7 cm

$\therefore$ Total surface area of the cylindrical part

$= 2 \times \frac{22}{7} \times \frac{7}{10}$ [2.4 + 0.7] cm^2

$= \frac{44}{10} \times 3.1$ cm^2 $= \frac{44 \times 31}{100} = \frac{1364}{100}$ cm^2

For conical part



Base radius (r) = 0.7 cm

and height (h) = 2.4 cm

$\therefore$ Slant height (ℓ) = $\sqrt{r^2 + h^2}$

$= \sqrt{(0.7)^2 + (2.4)^2}$

$= \sqrt{0.49 + 5.76} = \sqrt{6.25}$ = 2.5 cm

$\therefore$ Curved surface area of the conical part

$= \pi r \ell = \frac{22}{7} \times 0.7 \times 2.5$ cm^2 $= \frac{550}{100}$ cm^2

Base area of the conical part

$\pi r^2 = \frac{22}{7} \times \left(\frac{7}{10}\right)^2$ cm^2

$= \frac{22 \times 7}{100}$ cm^2 $= \frac{154}{100}$ cm^2

Total surface area of the remaining solid

= [(Total surface area of cylindrical part) + (Curved surface area of conical part) – (Base area of the conical part)]

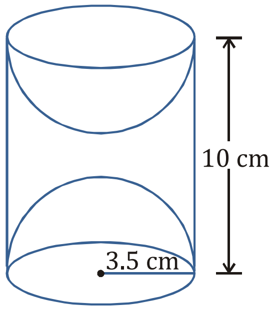
$\left(\frac{1364}{100} + \frac{550}{100} - \frac{154}{100}\right) = \frac{1760}{100}$ cm^2

= 17.6 cm^2.

Hence, total surface area to the nearest cm^2 is 18 cm^2.

9. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in fig. If the height of the cylinder is 10 cm, and its base is of radius 3·5 cm, find the total surface area of the article.

Sol.



Radius of the cylinder (r) = 3.5 cm

Height of the cylinder (h) = 10 cm

$\therefore$ Curved surface area of cylinder = $2\pi rh$

$= 2 \times \frac{22}{7} \times \frac{35}{10} \times 10$ cm^2

= 220 cm^2

Curved surface area of a hemisphere = $2\pi r^2$

$\therefore$ Curved surface area of both hemispheres= $2 \times 2\pi r^2 = 4\pi r^2$

$= 4 \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10}$ cm^2

= 154 cm^2

Total surface area of the remaining solid

= (Curved surface area of cylinder + curved surface area of 2 hemispheres)

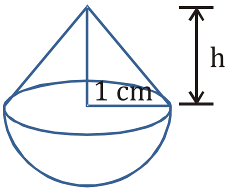
= (220 + 154) cm^2 = 374 cm^2.

EXERCISE : 12.2

Unless stated otherwise, take $\pi = \frac{22}{7}$

1. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.

Sol.



Here, r = 1 cm and h = 1 cm.

Volume of the conical part $= \frac{1}{3}\pi r^2 h$ and

volume of the hemispherical part $\frac{2}{3}\pi r^3$

$\therefore$ Volume of the solid shape

= (Volume of cone + Volume of hemisphere)

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2 (h + 2r)$

$= \frac{1}{3}\pi (1)^2 [1 + 2(1)]\ cm^3$

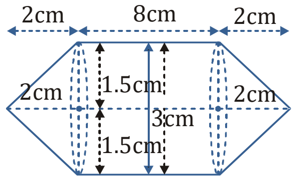
$= \frac{1}{3}\pi \times 1 \times 3\ cm^3$

$= \pi\ cm^3$

2. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same).

Sol. Volume of the cylindrical part

$= \pi \times (1.5)^2 \times 8\ cm^3 = 18\pi\ cm^3$



Volume of each conical part

$= \frac{1}{3}\pi \times (1.5)^2 \times 2\ cm^3 = \frac{3}{2}\pi\ cm^3$

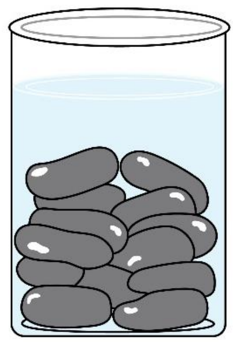
Therefore, the volume of the air

= The volume of cylindrical part + The volumes of two conical parts

$= 18\pi + 2 \times \frac{3}{2}\pi\ cm^3 = 21\pi\ cm^3$

$= 21 \times \frac{22}{7}\ cm^3 = 66\ cm^3$

3. A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm. (see fig.)

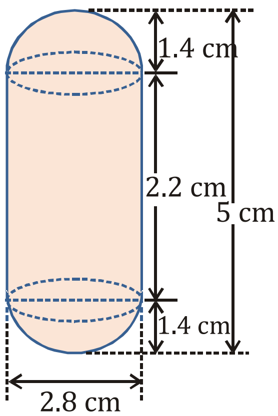


Sol. Since, a gulab jamun is like a cylinder with hemispherical ends.

Total height of the gulab jamun = 5 cm.

Diameter = 2.8 cm

$\Rightarrow$ Radius = 1.4 cm



$\therefore$ Length (height) of the cylindrical part

= 5cm – (1.4 + 1.4) cm

= 5 cm – 2.8 cm = 2.2 cm

Now, volume of the cylindrical part = $\pi r^2 h$ and volume of both the hemispherical ends = $2\left(\frac{2}{3}\pi r^3\right) = \frac{4}{3}\pi r^3$

$\therefore$ Volume of a gulab jamun

$= \pi r^2 h + \frac{4}{3}\pi r^3 = \pi r^2\left(h + \frac{4}{3}r\right)$

$= \frac{22}{7} \times (1.4)^2 \left[2.2 + \frac{4}{3}(1.4)\right] \text{cm}^3$

$= \frac{22}{7} \times \frac{14}{10} \times \frac{14}{10}\left(\frac{22}{10} + \frac{56}{30}\right) \text{cm}^3$

$= \frac{22}{7} \times \frac{14}{10} \times \frac{14}{10}\left(\frac{22}{10} + \frac{56}{30}\right) \text{cm}^2$

$= \frac{44 \times 14}{100} \times \frac{122}{30} \text{cm}^3$

Volume of 45 gulab jamuns

$= 45 \times \left(\frac{44 \times 14}{100} \times \frac{122}{30}\right) \text{cm}^3$

$= \frac{15 \times 44 \times 14 \times 122}{1000} \text{cm}^3$

Since, the quantity of syrup in gulab jamuns = 30% of [volume]

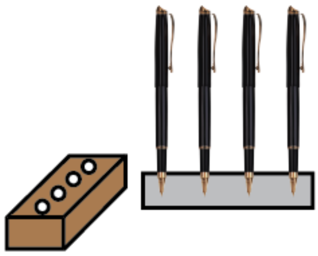
$= 30\% \text{ of } \left(\frac{15 \times 44 \times 14 \times 122}{1000}\right) \text{cm}^3$

$= \frac{30}{100} \times \frac{15 \times 44 \times 14 \times 122}{1000} \text{cm}^3$

= 338.184 cm^3

= 338 cm^3 (approx)

4. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see fig.).



Sol. Radius of conical cavity = 0.5 cm and depth (i.e., vertical height) = 1.4 cm

Volume of wood taken out to make one cavity

$= \frac{1}{3}\pi r^2 \times h = \frac{1}{3} \times \frac{22}{7} \times (0.5)^2 \times (1.4) \text{ cm}^3$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{1}{4} \times \frac{14}{10} \text{ cm}^3 = \frac{11}{30} \text{ cm}^3$

Volume of wood taken out to make four cavities = $4 \times \frac{11}{30} \text{ cm}^3 = \frac{44}{30} \text{ cm}^3$

Volume of the wood in the pen stand

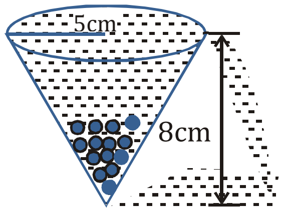
= Volume of cuboid – Volume of four cavities = $(15 \times 10 \times 3.5) - \frac{44}{30} \text{ cm}^3$

= (525 – 1.47) cm^3 (approx.)

= 523.53 cm^3 (approx.)

5. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water upto the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Sol.



Height of the conical vessel (h) = 8 cm

Base radius (r) = 5 cm

Volume of water in conical vessel = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 8 \text{ cm}^3$

$= \frac{4400}{21} \text{cm}^3$

Now, Total volume of lead shots

$= \frac{1}{4} \times \frac{4400}{21} \text{cm}^3 = \frac{1100}{21} \text{cm}^3$

Since, radius of spherical lead shot (r) = 0.5 cm

$\therefore$ Volume of 1 lead shot = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{5}{10} \times \frac{5}{10} \times \frac{5}{10} \text{cm}^3$

$\therefore$ Number of lead shots =

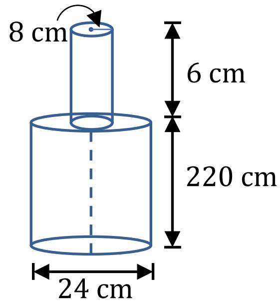
$\frac{\text{Total volume of lead shots}}{\text{Volume of 1 lead shot}}$

$= \frac{\left(\frac{1100}{21}\right)}{\left(\frac{4 \times 22 \times 5 \times 5 \times 5}{3 \times 7 \times 1000}\right)} = 100$

Thus, the required number of lead shots = 100.

6. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass. (Use $\pi = 3.14$)

Sol.



First cylindrical part has height 220 cm and radius 12 cm.

Its volume = $\pi \times (12)^2 \times 220 \text{ cm}^3$.

Second cylindrical part has height 60 cm and radius 8 cm.

Its volume = $\pi \times (8)^2 \times 60 \text{ cm}^3$

Total volume = $\{144 \times 220 + 64 \times 60\} \pi \text{ cm}^3$

$= 35520 \pi \text{ cm}^3 = 35520 \times 3.14 \text{ cm}^3$

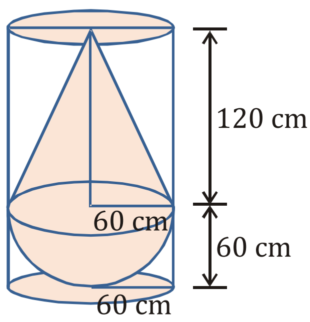
$= 111532.8 \text{ cm}^3$

Total weight (at the rate of 8 gm per 1 cm^3)

$= \frac{111532.8 \times 8}{1000} \text{kg} = 111.5328 \times 8 \text{ kg}$

$= 892.2624 \text{ kg}$

7. A solid consisting of a right circular cone of height120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius is 60 cm and its height is 180 cm.



Sol. Height of the conical part = 120 cm

Base radius of the conical part = 60 cm

$\therefore$ Volume of the conical part = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\times\frac{22}{7}\times 60^2 \times 120\ cm^3$

Radius of the hemispherical part = 60 cm.

$\therefore$ Volume of the hemispherical part = $\frac{2}{3}\pi r^3$

$= \frac{2}{3}\times\frac{22}{7}\times (60)^3\ cm^3$

$\therefore$ Volume of the solid = [Volume of conical part] + [Volume of hemispherical part]

$= \left(\frac{1}{3}\times\frac{22}{7}\times 60^2\times 120\right)+\left(\frac{2}{3}\times\frac{22}{7}\times 60^3\right) cm^3$

$= \frac{2}{3}\times\frac{22}{7}\times 60^2\ [60 + 60]\ cm^3$

$= \frac{2}{3}\times\frac{22}{7}\times 60 \times 60 \times 120\ cm^3$

$= \frac{6336000}{7}\ cm^3$

$\Rightarrow$ Volume of water in the cylinder = $\pi r^2 h$

$= \frac{22}{7}\times 60\times 60 \times 180 = \frac{14256000}{7}\ cm^3$

$\therefore$ Volume of water left in the cylinder

= Volume of cylinder – Volume of solid

$= \left(\frac{14256000}{7}-\frac{6336000}{7}\right) cm^3$

$= \frac{7920000}{7}\ cm^3$

$= 1131428.57142\ cm^3$

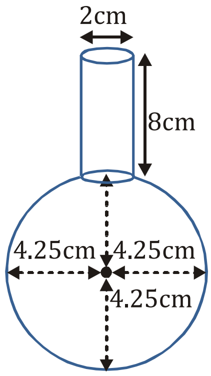
$= \frac{1131428.57142}{1000000}\ m^3$

$= 1.13142857142\ m^3$

$= 1.131\ m^3$ (approx.)

8. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm³. Check whether she is correct, taking the above as the inside measurements, and π = 3.14.

Sol.



The cylinder neck has length = 8 cm

and radius = 1 cm

Volume of the cylinder part

$= \pi (1)^2 \times 8\ cm^3$

$= 8\pi\ cm^3$

The radius of the spherical part

$= \frac{8.5}{2} cm = 4.25 cm$

Volume of the spherical part

$= \frac{4}{3}\pi \times (4.25)^3$

Total volume of water

= Volume of cylindrical part + volume of spherical part.

$= 8\pi + \frac{4}{3}\times (4.25)^3\ \pi\ cm^3$

$= 8 \times 3.14 + \frac{4}{3}\times 3.14 \times (4.25)^3\ cm^3$

= 25.12 + 321.38 (approx.)

$= 346.5\ cm^3$ (approx.)

So, $345\ cm^3$ is not correct.

Deleted Exercise

Take $\pi = \frac{22}{7}$, unless stated otherwise.

1. A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Sol. Let the height of the cylinder made be equal to h cm.

Its radius = 6 cm

Volume of the cylinder made = Volume of the given metallic sphere of radius 4.2 cm

$\Rightarrow \quad \pi \times (6)^2 \times h = \frac{4}{3} \pi \times (4.2)^3 \text{ cm}^3$

$\Rightarrow \quad 36 \times h = \frac{4}{3} \times 4.2 \times 4.2 \times 4.2$

$\Rightarrow \quad h = \frac{4.2 \times 4.2 \times 4.2}{3 \times 9} \text{ cm}$

$= 1.4 \times 1.4 \times 1.4 \text{ cm}$

$= 2.744 \text{ cm}$

2. Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Sol. Radii of the given spheres are

$r_1 = 6 \text{ cm}, r_2 = 8 \text{ cm}, r_3 = 10 \text{ cm}$

$\Rightarrow$ Volume of the given spheres are

$V_1 = \frac{4}{3}\pi r_1^3, V_2 = \frac{4}{3}\pi r_2^3 \text{ and } V_3 = \frac{4}{3}\pi r_3^3$

$\therefore$ Total volume of the given spheres

$= V_1 + V_2 + V_3$

$= \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3$

$= \frac{4}{3}\pi(r_1^3 + r_2^3 + r_3^3)$

$= \frac{4}{3} \times \frac{22}{7} \times (6^3 + 8^3 + 10^3) \text{ cm}^3$

$= \frac{4}{3} \times \frac{22}{7} \times (216 + 512 + 1000) \text{ cm}^3$

$= \frac{4}{3} \times \frac{22}{7} \times 1728 \text{ cm}^3$

Let the radius of the new big sphere be R.

$\therefore$ Volume of the new sphere = $\frac{4}{3} \times \pi \times R^3$

Since, the two volume must be equal.

$\therefore \frac{4}{3} \times \frac{22}{7} \times R^3 = \frac{4}{3} \times \frac{22}{7} \times 1728 \text{ cm}^3$

$\Rightarrow R^3 = 1728 \Rightarrow R = 12 \text{ cm}$

Thus, the required radius of the resulting sphere = 12 cm.

3. A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

Sol. Diameter of the cylindrical well = 7 m

$\Rightarrow$ Radius of the cylindrical (r) = $\frac{7}{2}$ m

Depth of the well (h) = 20 m

$\therefore$ Volume = $\pi r^2 h$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20 \text{ m}^2 = 22 \times 7 \times 5 \text{ m}^3$

$\Rightarrow$ Volume of the earth taken out = $22 \times 7 \times 5 \text{ m}^3$

Now this earth is spread out to form a cuboidal platform having length = 22 m, breadth = 14 m

Let h be the height of the platform.

$\therefore$ Volume of the platform = $22 \times 14 \times h \text{ m}^3$

$\therefore \quad 22 \times 14 \times h = 22 \times 7 \times 5$

$\Rightarrow \quad h = \frac{22 \times 7 \times 5}{22 \times 14} = \frac{5}{2} \text{ m} = 2.5 \text{ m}$

Thus, the required height of the platform is 2.5 m

4. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

Sol. Diameter of cylindrical well (d) = 3 m

$\Rightarrow$ Radius of the cylindrical well = $\frac{3}{2}$ m

= 1.5m

Depth of the well (h) = 14 m

$\therefore$ Volume = $\pi r^2 h = \frac{22}{7} \times \left(\frac{15}{10}\right)^2 \times 14\ m^3$

$= \frac{22 \times 15 \times 15 \times 14}{7 \times 10 \times 10} m^3 = 99\ m^3$

Let the height of the embankment = H metre.

Internal radius of the embankment (r) = 1.5 m.

External radius of the embankment

R = (4 + 1.5) m = 5.5 m.

$\therefore$ Volume of the embankment

$= \pi R^2 H - \pi r^2 H = \pi H[R^2 - r^2]$

$= \pi H(R + r)(R - r)$

$= \frac{22}{7} \times H(5.5 + 1.5)(5.5 - 1.5)$

$= \frac{22}{7} \times H \times 7 \times 4 m^3$

Since, Volume of the embankment = Volume of the cylindrical well

$\therefore \left[\frac{22}{7} \times H \times 7 \times 4\right] = 99$

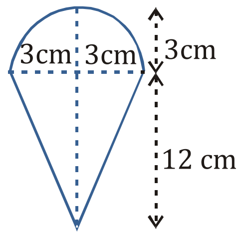
$\Rightarrow H = 99 \times \frac{7}{22} \times \frac{1}{7} \times \frac{1}{4} m = \frac{9}{8} m$

= 1.125 m

Thus, the required height of the embankment = 1.125 m

5. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice-cream. The ice-cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice-cream.

Sol.



Volume of one ice-cream cone as shown in figure.

$= \frac{1}{3} \pi \times (3)^2 \times 12 + \frac{2}{3} \pi \times (3)^3\ cm^3$

$= 36\pi + 18\pi = 54\pi\ cm^3$

volume of the ice-cream in the cylindrical container (of height 15 cm and diameter 12 cm) = $\pi \times (6)^2 \times 15$

Let the number of cones made be n.

Then, $n \times 54\pi = \pi \times (6)^2 \times 15$

$\Rightarrow 54n = 36 \times 15 \Rightarrow n = 10$

$\therefore$ Required no. of cones = 10

6. How many silver coins 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm?

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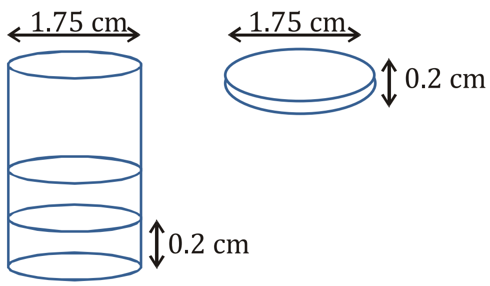
Sol.



For a circular coin

Diameter = 1.75 cm

$\Rightarrow$ Radius (r) = $\frac{175}{200}$ cm

Thickness (h) = 2mm = $\frac{2}{10}$ cm

$\therefore$ Volume = $\pi r^2 h$

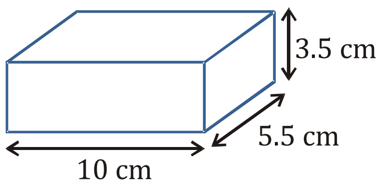
$$= \frac{22}{7} \times \left(\frac{175}{200}\right)^2 \times \frac{2}{10} \text{ cm}^3$$

For a cuboid :

Length (ℓ) = 10 cm,

Breadth (b) = 5.5 cm

Height (h) = 3.5 cm



$\therefore$ Volume = $\ell \times b \times h$

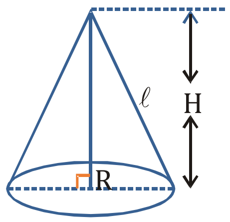
$$= 10 \times \frac{55}{10} \times \frac{35}{10} \text{ cm}^3$$

Number of coins = $\frac{\text{Volume of cuboid}}{\text{Volume of one coin}}$

$$= \frac{10 \times \frac{55}{10} \times \frac{35}{10}}{\frac{22}{7} \times \left(\frac{175}{200}\right)^2 \times \frac{2}{10}} = 400$$

Thus, the required number of coins = 400.

7. A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.



Sol. For the cylindrical bucket

Radius (r) = 18 cm and height (h) = 32 cm

Volume = $\pi r^2 h = \frac{22}{7}(18)^2 \times 32$ cm^3

$\Rightarrow$ Volume of the sand

$$= \left(\frac{22}{7} \times 18 \times 18 \times 32\right) \text{cm}^3$$

For the conical heap

Height (H) = 24 cm & let radius of the base be (R).

$\therefore$ Volume of conical heap

$$= \frac{1}{3}\pi R^2 H = \left(\frac{1}{3} \times \frac{22}{7} \times R^2 \times 24\right) \text{cm}^3$$

$\because$ Volume of conical heap of the sand = Volume of the sand

$$\therefore \frac{1}{3} \times \frac{22}{7} \times R^2 \times 24 = \frac{22}{7} \times 18 \times 18 \times 32$$

$$\Rightarrow R^2 = \frac{18 \times 18 \times 32 \times 3}{24} = 18^2 \times 2^2$$

$$\Rightarrow R = \sqrt{18^2 \times 2^2} = 18 \times 2 \text{ cm} = 36 \text{ cm}$$

Let 'ℓ' be the slant height of the conical heap of the sand.

$\therefore \quad \ell^2 = H^2 + R^2$

$\Rightarrow \quad \ell^2 = 24^2 + 36^2$

$\Rightarrow \quad \ell^2 = (12 \times 2)^2 + (12 \times 3)^2$

$\Rightarrow \quad \ell^2 = 12^2[2^2 + 3^2]$

$\Rightarrow \quad \ell^2 = 12^2 \times 13$

$\Rightarrow \quad \ell = \sqrt{12^2 \times 13} = 12 \times \sqrt{13}$

Thus, the required radius = 36 cm and slant height = $12\sqrt{13}$ cm

8. Water in a canal 6 m wide and 1.5 m deep is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

Sol. Depth of water in the canal = 1.5 m (height)

Width of canal = 6 m

Length of canal = 10 × 1000 m

Volume of water flowing through the canal in 60 minutes = (10 × 1000) × 6 × 1.5 cm^3

($\because$ Speed = 10 km/hr = 10 × 1000 m per hr)

Volume of water flowing through canal in 30 minutes

$= 10000 \times 9 \times \frac{30}{60} m^3 = 45000\ m^3$

Let the required area be x m^2.

Then $x \times \frac{8}{100} = 45000$

($\because$ 8 cm deep water = $\frac{8}{100}$ m deep)

$\Rightarrow \quad x = \frac{4500000}{8} m^2 = 562500\ m^2$

$= \frac{562500}{10000}$ hectares = 56.25 hectares

9. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

Sol. Water speed = 3 km/hr

$= \frac{3000}{60} m/min$

= 50 m/min

Diameter of the pipe = 20 cm

i.e., radius = 10 cm = $\frac{1}{10}$ m

Water tank has 2m depth and 10 m diameter, i.e., radius 5 m

Let the required time to fill the tank be n minutes.

Then water flowing through the pipe in n minutes = Volume of the water tank.

$\Rightarrow \quad \pi \times \left(\frac{1}{10}\right)^2 \times \{n \times 50\} = \pi \times (5)^2 \times 2$

$\frac{1}{100} \times n \times 50 = 50 \Rightarrow n = 100$

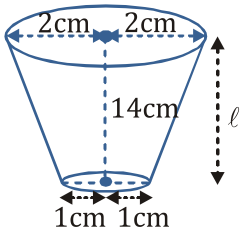
Hence, the required time is 100 minutes.

EXERCISE : 12.4

Use $\pi = \frac{22}{7}$, unless stated otherwise.

1. A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.

Sol.



R = 2 cm, r = 1 cm, h = 14 cm

Capacity of the glass = volume of the frustum with radii of ends as 2 cm and 1 cm and height 14 cm

$= \frac{1}{3}\pi h \{R^2 + r^2 + Rr\}$

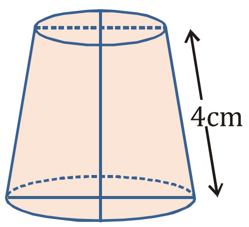
$= \frac{1}{3}\pi \times 14 \times \{(2)^2 + (1)^2 + 2(1)\}\ cm^3$

$= \frac{1}{3} \times \frac{22}{7} \times 14 \times 7\ cm^3$

$= \frac{308}{3} = 102\frac{2}{3} cm^3$

2. The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Sol. We have, Slant height (ℓ) = 4 cm



$2\pi r_1 = 18$ cm and $2\pi r_2 = 6$ cm

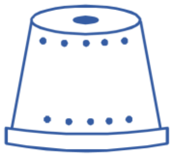
$\Rightarrow \quad \pi r_1 = \frac{18}{2} = 9$ cm and $\pi r_2 = \frac{6}{2} = 3$ cm

$\therefore$ Curved surface area of the frustum of the cone $= \pi(r_1 + r_2)\ \ell = (\pi r_1 + \pi r_2)\ \ell$

$= (9 + 3) \times 4\ cm^2$

$= 12 \times 4\ cm^2 = 48\ cm^2$.

3. A fez, the cap used by the Turks, is shaped like the frustum of a cone (see fig.). If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 15 cm, find the area of material used for making it.



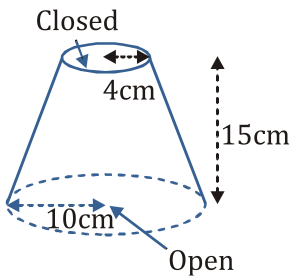
Sol. R = 10 cm, r = 4 cm, ℓ = 15 cm

Curved surface area $= \pi \times \ell \times \{R + r\}$

$= \pi \times 15 \times \{10 + 4\}\ cm^2$

$= \frac{22}{7} \times 15 \times 14\ cm^2 = 660\ cm^2$

Area of the closed side



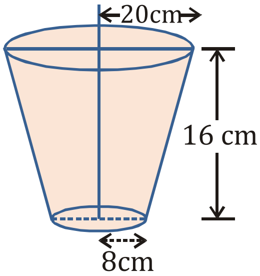
$= \pi r^2 = \frac{22}{7} \times (4)^2 = \frac{352}{7} = 50\frac{2}{7}\ cm^2$

total area of the material used = curved surface area of cap + area of base

$= (660 + 50\frac{2}{7})\ cm^2 = 710\frac{2}{7}\ cm^2$

4. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of Rs. 20 per litre. Also find the cost of metal sheet used to make the container, if it costs Rs. 8 per 100 cm². (Take π = 3.14).

Sol.



We have : r_1 = 20 cm,

r_2 = 8 cm and h = 16 cm

∴ Volume of the frustum

$= \frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1r_2]$

$= \frac{1}{3} \times \frac{314}{100} \times 16[20^2 + 8^2 + 20 \times 8]\ cm^3$

$= \frac{1}{3} \times \frac{314}{100} \times 16 \times [400 + 64 + 160]\ cm^3$

$= \frac{1}{3} \times \frac{314}{100} \times 16 \times 624\ cm^3$

$= \left[\frac{314}{100} \times 16 \times 208\right] cm^3$

$= \left[\frac{314}{100} \times 16 \times 208\right] \div 1000$ litres

$= \frac{314 \times 16 \times 208}{100000}$ litres

∴ Cost of milk = ₹ 20 × $\frac{314 \times 16 \times 208}{100000}$ litres = ₹ 208.998 ≈ 209

Now, slant height of the given frustum

$\ell = \sqrt{h^2 + (r_1 - r_2)^2} = \sqrt{16^2 + (20-8)^2}$

$= \sqrt{16^2 + 12^2} = \sqrt{256 + 144}$

$= \sqrt{400} = 20$ cm

∴ Curved surface area = $\pi(r_1 + r_2)\ell$

= 3.14 × (20 + 8) × 20 cm^2

$= \frac{314}{100} \times 28 \times 20\ cm^2 = 1758.4\ cm^2$

Area of the bottom = πr_2^2

$= \frac{314}{100} \times 8 \times 8\ cm^2 = 200.96\ cm^2$

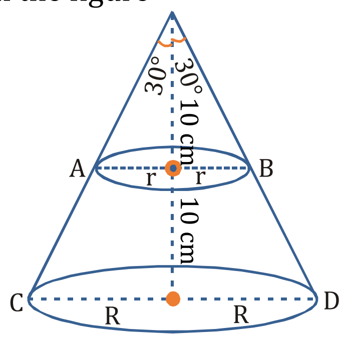
∴ Total area of metal required = CSA + area of bottom

= 1758.4 cm^2 + 200.96 cm^2 = 1959.36 cm^2

Cost of metal required = ₹ $\frac{8}{100}$ × 1959.36 cm^2 = ₹ 156.75

5. A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $\left(\frac{1}{16}\right)$ cm, find the length of the wire.

Sol. From the figure



$\frac{R}{20} = \tan 30° = \frac{1}{\sqrt{3}}$, i.e., $R = \frac{20}{\sqrt{3}}$ cm

$\frac{r}{10} = \tan 30° = \frac{1}{\sqrt{3}}$, i.e., $r = \frac{10}{\sqrt{3}}$ cm

h = 10 cm is the height of the frustum.

Volume of the material in the frustum ACDB

$= \frac{1}{3}\pi \times h \times (R^2 + r^2 + Rr)$

$= \frac{1}{3}\pi \times 10 \times \left(\frac{400}{3} + \frac{100}{3} + \frac{200}{3}\right) cm^3$

$= \frac{7000}{9}\pi cm^3$

Now let us suppose wire of diameter $\frac{1}{16}$ cm is made of length x cm.

Then, $\pi \times \left(\frac{1}{32}\right)^2 \times x = \frac{7000}{9}\pi$

$x = \frac{7000 \times 32 \times 32}{9}$ cm

$x = \frac{71680}{9}$ m = 7964.4 m

∴ Required length of wire = 7964 m