

## NCERT QUESTIONS WITH SOLUTIONS

### EXERCISE : 1.1

1. Use Euclid's division algorithm to find the HCF of:

(i) 135 and 225

(ii) 196 and 38220

(iii) 867 and 225.

**Sol.** (i) 135 and 225. Start with the larger integer, that is, 225. Apply the division lemma to 225 and 135, to get.

$$225 = 135 \times 1 + 90$$

Since the remainder  $90 \neq 0$ , we apply the division lemma to 135 and 90 to get

$$135 = 90 \times 1 + 45$$

We consider the new divisor 90 and the new remainder 45, and apply the division lemma to get

$$90 = 45 \times 2 + 0$$

The remainder has now become zero, so our procedure stops.

Since the divisor at this stage is 45, the HCF of 225 and 135 is 45.

(ii) 196 and 38220

Start with the larger integer, that is, 38220. Apply the division lemma to 38220 and 196, to get

$$38220 = 196 \times 195 + 0$$

Remainder at this stage is zero, so our procedure stops.

So, HCF of 196 and 38220 is 196.

(iii) 867 and 225

Start with the larger integer, that is, 867. Apply the division lemma to 867 and 225, to get

$$867 = 225 \times 3 + 192$$

Since the remainder  $192 \neq 0$ , we apply the division lemma to 225 and 192 to get

$$225 = 192 \times 1 + 33$$

Since the remainder  $33 \neq 0$ , we apply the division lemma to 192 and 33 to get

$$192 = 33 \times 5 + 27$$

Since the remainder  $27 \neq 0$ , we apply the division lemma to 33 and 27 to get

$$33 = 27 \times 1 + 6$$

Since the remainder  $6 \neq 0$ , we apply the division lemma to 27 and 6 to get

$$27 = 6 \times 4 + 3$$

Since the remainder  $3 \neq 0$ , we apply the division lemma to 6 and 3 to get

$$6 = 3 \times 2 + 0$$

Now, remainder at this stage is zero, so our procedure stops.

So, HCF of 867 and 225 is 3.

2. Show that any positive odd integer is of the form  $6q + 1$ , or  $6q + 3$  or  $6q + 5$ , where  $q$  is some integer.

**Sol.** Let us start with taking  $a$ , where  $a$  is any positive odd integer. We apply the division algorithm, with  $a$  and  $b = 6$ . Since  $0 \leq r < 6$ , the possible remainders are 0, 1, 2, 3, 4, 5. That is,  $a$  can be  $6q$  or  $6q + 1$ , or

$6q + 2$ , or  $6q + 3$ , or  $6q + 4$ , or  $6q + 5$ , where  $q$  is the quotient. However, since  $a$  is odd, we do not consider the cases  $6q$ ,  $6q + 2$  and  $6q + 4$  (since all the three are divisible by 2). Therefore, any positive odd integer is of the form  $6q + 1$  or  $6q + 3$  or  $6q + 5$ .

3. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

**Sol.** Let  $a = 616$ ,  $b = 32$  by Euclid division algorithm

$$616 = 32 \times 19 + 8$$

$$32 = 8 \times 4 + 0$$

$$\text{HCF of } (616, 32) = 8$$

4. Use Euclid's division lemma to show that the square of any positive integer is either of the form  $3m$  or  $3m + 1$  for some integer  $m$ .

**Sol.** Let  $a$  be any positive integer. We apply the division lemma with  $b = 3$ . Since  $0 \leq r < 3$ , the possible remainders are 0, 1 and 2. That is,  $a$  can be  $3q$ , or  $3q + 1$ , or  $3q + 2$ , where  $q$  is the quotient.

$$\text{Now, } (3q)^2 = 9q^2 = 3 \times 3q^2$$

which can be written in the form  $3m$ , where  $m = 3q^2$ , since 9 is divisible by 3.

$$\begin{aligned} \text{Again, } (3q + 1)^2 &= 9q^2 + 6q + 1 \\ &= 3(3q^2 + 2q) + 1 \end{aligned}$$

which can be written in the form  $3m + 1$ , where  $m = 3q^2 + 2q$ , since  $9q^2 + 6q$ , i.e.,  $3(3q^2 + 2q)$  is divisible by 3.

Lastly,

$$\begin{aligned} (3q + 2)^2 &= 9q^2 + 12q + 4 \\ &= (9q^2 + 12q + 3) + 1 \\ &= 3(3q^2 + 4q + 1) + 1 \end{aligned}$$

which can be written in the form  $3m + 1$ , where  $m$

$$\begin{aligned} &= 3q^2 + 4q + 1, \text{ since } 9q^2 + 12q + 3, \\ \text{i.e., } &3(3q^2 + 4q + 1) \text{ is divisible by 3.} \end{aligned}$$

Therefore, the square of any positive integer is either of the form  $3m$  or  $3m + 1$  for some integer  $m$ .

5. Use Euclid's division lemma to show that the cube of any positive integer is of the form  $9m$ ,  $9m + 1$  or  $9m + 8$ .

**Sol.** Let  $n$  be any positive integer. We apply the division lemma with  $b = 3$ . Since  $0 \leq r < 3$ , the possible remainders are 0, 1 and 2. That is,  $n$  can be  $3q$ , or  $3q + 1$ , or  $3q + 2$ , where  $q$  is the quotient.

**Case I :** Let  $n = 3q$

Cube of this will be

$$n^3 = 27q^3$$

$$n^3 = 9(3q^3)$$

$$\text{So, } n^3 = 9m, \text{ where } m = 3q^3.$$

**Case II :**  $n = 3q + 1$

$$\text{So, } n^3 = (3q + 1)^3$$

$$n^3 = 27q^3 + 1 + 27q^2 + 9q$$

$$n^3 = 9(3q^3 + 3q^2 + q) + 1$$

$$n^3 = 9m + 1, \text{ where } m = 3q^3 + 3q^2 + q$$

**Case III :**  $n = 3q + 2$

So,  $n^3 = (3q + 2)^3$

$n^3 = 27q^3 + 54q^2 + 36q + 8$

$n^3 = 9(3q^3 + 6q^2 + 4q) + 8$

$n^3 = 9m + 8$ , where  $m = 3q^3 + 6q^2 + 4q$

So, it means cube of positive integer is of the form  $9m, 9m + 1$  or  $9m + 8$ .

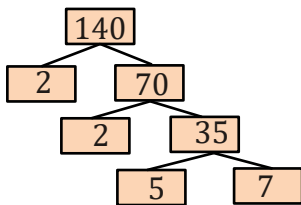
**EXERCISE : 1.2**

1. Express each number as product of its prime factors :

(i) 140    (ii) 156    (iii) 3825

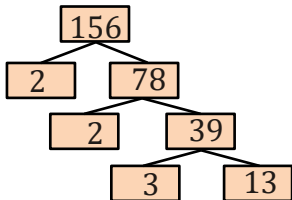
(iv) 5005    (v) 7429

**Sol.** (i) 140



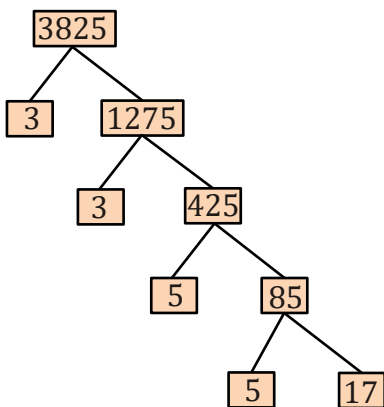
So,  $140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$

(ii) 156



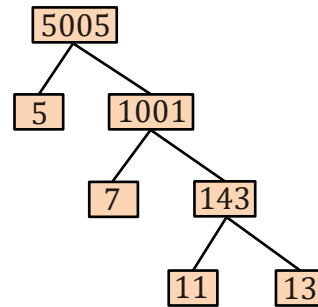
So,  $156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$

(iii) 3825



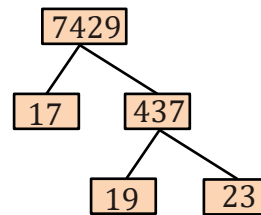
So,  $3825 = 3^2 \times 5^2 \times 17$

(iv) 5005



So,  $5005 = 5 \times 7 \times 11 \times 13$

(v) 7429



So,  $7429 = 17 \times 19 \times 23$

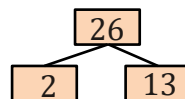
2. Find the LCM and HCF of the following pairs of integers and verify that  $LCM \times HCF = \text{product of two numbers}$ .

(i) 26 and 91

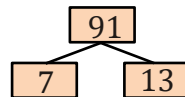
(ii) 510 and 92

(iii) 336 and 54

**Sol.** (i) 26 and 91



So,  $26 = 2 \times 13$



So,  $91 = 7 \times 13$

Therefore,

$LCM(26, 91) = 2 \times 7 \times 13 = 182$

$HCF(26, 91) = 13$

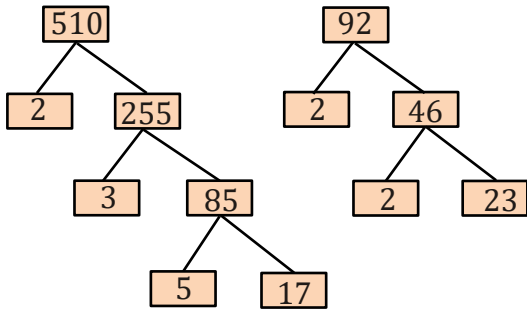
Verification :

$$\text{LCM} \times \text{HCF} = 182 \times 13 = 2366$$

$$\text{and } 26 \times 91 = 2366$$

i.e.,  $\text{LCM} \times \text{HCF} = \text{product of two numbers.}$

(ii) 510 and 92



$$510 = 2 \times 3 \times 5 \times 17$$

$$92 = 2^2 \times 23$$

$$\text{LCM}(510, 92) = 2^2 \times 3 \times 5 \times 17 \times 23 = 23460$$

$$\text{HCF}(510, 92) = 2$$

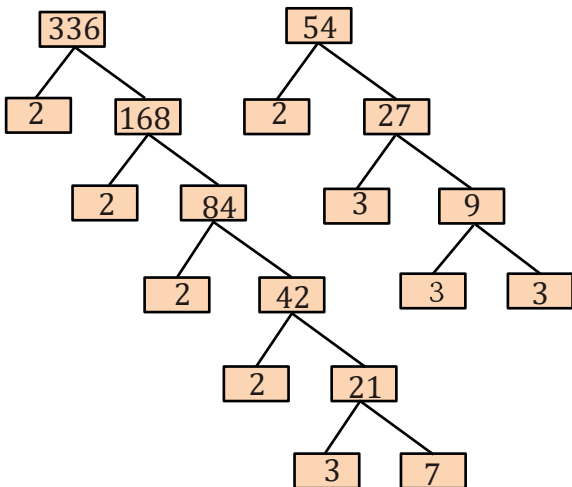
Verification :

$$\text{LCM} \times \text{HCF} = 23460 \times 2 = 46920$$

$$\text{and } 510 \times 92 = 46920$$

i.e.,  $\text{LCM} \times \text{HCF} = \text{product of two numbers.}$

(iii) 336 and 54



$$336 = 2^4 \times 3 \times 7 \text{ and } 54 = 2 \times 3^3$$

$$\text{LCM} = 2^4 \times 3^3 \times 7 = 3024$$

$$\text{HCF} = 2 \times 3 = 6$$

Verification,

$$\text{LCM} \times \text{HCF} = 2^4 \times 3^3 \times 7 \times 2 \times 3 = 18144$$

$$\text{Product of two numbers} = 336 \times 54 = 18144$$

i.e.,  $\text{LCM} \times \text{HCF} = \text{product of two numbers.}$

3. Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21

(ii) 17, 23 and 29

(iii) 8, 9 and 25

**Sol.** (i) 12, 15 and 21

$$\text{So, } 12 = 2 \times 2 \times 3 = 2^2 \times 3$$

$$\text{So, } 15 = 3 \times 5$$

$$\text{So, } 21 = 3 \times 7$$

Therefore,

$$\text{HCF}(12, 15, 21) = 3;$$

$$\text{LCM} = (12, 15, 21) = 2^2 \times 3 \times 5 \times 7 = 420$$

(ii) 17, 23, 29

$$17 = 1 \times 17$$

$$23 = 1 \times 23$$

$$29 = 1 \times 29$$

$$\text{LCM} = 1 \times 17 \times 23 \times 29 = 11339$$

$$\text{HCF} = 1$$

(iii) 8, 9, 25

$$8 = 2 \times 2 \times 2$$

$$9 = 3 \times 3$$

$$25 = 5 \times 5$$

$$\text{LCM} = 2^3 \times 3^2 \times 5^2 = 1800$$

$$\text{HCF} = 1$$

4. Given that  $\text{HCF}(306, 657) = 9$ , find  $\text{LCM}(306, 657)$ .

**Sol.** i.e.,  $\text{LCM} \times \text{HCF} = \text{product of two numbers}$ .

$$\begin{aligned} \text{LCM}(306, 657) &= \frac{306 \times 657}{\text{HCF}(306, 657)} \\ &= \frac{306 \times 657}{9} = 22338. \end{aligned}$$

5. Check whether  $6^n$  can end with the digit 0 for any natural number  $n$ .

**Sol.** If the number  $6^n$ , for any natural number  $n$ , ends with digit 0, then it would be divisible by 5. That is, the prime factorisation of  $6^n$  would contain the prime number 5. This is not possible because  $6^n = (2 \times 3)^n = 2^n \times 3^n$ ; so the only primes in the factorisation of  $6^n$  are 2 and 3 and the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorisation of  $6^n$ . So, there is no natural number  $n$  for which  $6^n$  ends with the digit zero.

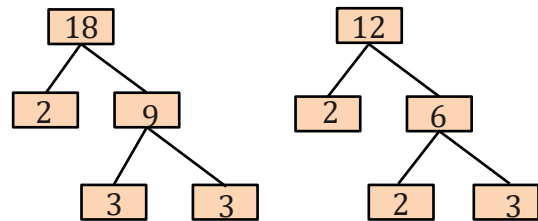
6. Explain why  $7 \times 11 \times 13 + 13$  and  $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$  are composite numbers.

**Sol.** (i) Let  $7 \times 11 \times 13 + 13 = (7 \times 11 + 1) \times 13 = (77 + 1) \times 13 = 78 \times 13$   
 $\Rightarrow 7 \times 11 \times 13 + 13 = 2 \times 3 \times 13 \times 13$   
 $= 2 \times 3 \times 13^2$  is a composite number as powers of prime occur and also having more than two distinct factors.

(ii)  $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1) \times 5 = 1009 \times 5$   
 Since,  $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$  can be expressed as a product of primes, other than 1 therefore it is a composite number.

7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

**Sol.** LCM of 18 and 12.



$$18 = 2 \times 3^2$$

$$12 = 2^2 \times 3$$

$$\text{LCM}(18, 12) = 2^2 \times 3^2 = 36$$

Thus, after 36 minutes they will meet again at the starting point.

**EXERCISE : 1.3**

1. Prove that  $\sqrt{5}$  is irrational.

**Sol.** Let us assume, to the contrary, that  $\sqrt{5}$  is rational.

So, we can find coprime integers  $a$  and  $b$  ( $b \neq 0$ ) such that

$$\sqrt{5} = \frac{a}{b}, b \neq 0, a, b \in I$$

$$\Rightarrow \sqrt{5} b = a$$

Squaring on both sides, we get

$$5b^2 = a^2$$

Therefore, 5 divides  $a^2$ .

(by fundamental theorem of arithmetic)

Therefore, 5 divides  $a$

So, we can write  $a = 5c$  for some integer  $c$ .

Substituting for a, we get

$$5b^2 = 25c^2$$

$$\Rightarrow b^2 = 5c^2$$

This means that 5 divides  $b^2$ , and so 5 divides b.

Therefore, a and b have at least 5 as a common factor.

But this contradicts the fact that a and b have no common factor other than 1.

This contradiction has arisen because of our incorrect assumption that  $\sqrt{5}$  is rational.

So, we conclude that  $\sqrt{5}$  is irrational.

2. Prove that  $3 + 2\sqrt{5}$  is irrational.

**Sol.** Let us assume, to the contrary, that  $3 + 2\sqrt{5}$  is rational. So, we can find coprime integers a and b ( $b \neq 0$ ) such that  $3 + 2\sqrt{5}$

$$= \frac{a}{b}, b \neq 0,$$

$$a, b \in I$$

$$\text{Therefore, } \frac{a}{b} - 3 = 2\sqrt{5}$$

$$\Rightarrow \frac{a-3b}{b} = 2\sqrt{5}$$

$$\Rightarrow \frac{a-3b}{2b} = \sqrt{5} \Rightarrow \frac{a}{2b} - \frac{3}{2} = \sqrt{5}$$

Since a and b are integers, we get  $\frac{a}{2b} - \frac{3}{2}$

is rational, and so  $\frac{a-3b}{2b} = \sqrt{5}$  is rational.

But this contradicts the fact that  $\sqrt{5}$  is irrational. This contradiction has arisen because of our incorrect assumption that  $3 + 2\sqrt{5}$  is rational.

So, we conclude that  $3 + 2\sqrt{5}$  is irrational.

3. Prove that the following are irrationals :

$$(i) \frac{1}{\sqrt{2}} \quad (ii) 7\sqrt{5} \quad (iii) 6 + \sqrt{2}$$

**Sol.** (i) Let us assume, to the contrary, that  $\frac{1}{\sqrt{2}}$  is rational. That is we can find

coprime integers p and q ( $q \neq 0$ ) such that,

$$\frac{1}{\sqrt{2}} = \frac{p}{q}, q \neq 0, p, q \in I$$

$$\text{Therefore, } \frac{q}{p} = \sqrt{2}$$

Since p and q are integers, we get  $\frac{q}{p}$  is

rational, and so  $\frac{q}{p} = \sqrt{2}$  is rational.

But this contradicts the fact that  $\sqrt{2}$  is irrational.

This contradiction has arisen because of our incorrect assumption that  $\frac{1}{\sqrt{2}}$  is

rational. So, we conclude that  $\frac{1}{\sqrt{2}}$  is

irrational.

(ii) Let us assume, to the contrary, that  $7\sqrt{5}$  is rational.

That is, we can find coprime integers a and b ( $b \neq 0$ ) such that  $7\sqrt{5} = \frac{a}{b}$ ,

$$b \neq 0, a, b \in I$$

$$\text{Therefore, } \frac{a}{7b} = \sqrt{5}$$

Since a and b are integers, we get  $\frac{a}{7b}$  is

rational, and so  $\frac{a}{7b} = \sqrt{5}$  is rational.

But this contradicts the fact that  $\sqrt{5}$  is irrational. This contradiction has arisen because of our incorrect assumption that  $7\sqrt{5}$  is rational.

So, we conclude that  $7\sqrt{5}$  is irrational.

(iii) Let us assume, to the contrary, that  $6 + \sqrt{2}$  is rational.

That is, we can find coprime integers  $a$  and  $b$  ( $b \neq 0$ ) such that  $6 + \sqrt{2} = \frac{a}{b}$

$$\text{Therefore, } \frac{a}{b} - 6 = \sqrt{2}$$

$$\Rightarrow \frac{a-6b}{b} = \sqrt{2}$$

Since  $a$  and  $b$  are integers, we get  $\frac{a}{b} - 6$  is

rational, and so  $\frac{a-6b}{b} = \sqrt{2}$  is rational.

But this contradicts the fact that  $\sqrt{2}$  is irrational. This contradiction has arisen because of our incorrect assumption that  $6 + \sqrt{2}$  is rational.

So, we conclude that  $6 + \sqrt{2}$  is irrational.

**EXERCISE : 1.4**

1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion.

(i)  $\frac{13}{3125}$

(ii)  $\frac{17}{8}$

(iii)  $\frac{64}{455}$

(iv)  $\frac{15}{1600}$

(v)  $\frac{29}{343}$

(vi)  $\frac{23}{2^3 5^2}$

(vii)  $\frac{129}{2^2 5^7 7^5}$

(viii)  $\frac{6}{15}$

(ix)  $\frac{35}{50}$

(x)  $\frac{77}{210}$

**Sol.** (i)  $\frac{13}{3125} = \frac{13}{5^5}$

Hence,  $q = 5^5$ , which is of the form  $2^n 5^m$  ( $n = 0, m = 5$ ). So, the rational number

$$\frac{13}{3125}$$

has a terminating decimal expansion.

(ii)  $\frac{17}{8} = \frac{17}{2^3}$

Hence,  $q = 2^3$ , which is of the form  $2^n 5^m$  ( $n = 3, m = 0$ ). So, the rational number

$$\frac{17}{8}$$

has a terminating decimal expansion.

(iii)  $\frac{64}{455} = \frac{64}{5 \times 7 \times 13}$

Hence,  $q = 5 \times 7 \times 13$ , which is not of the form  $2^n 5^m$ . So, the rational number  $\frac{64}{455}$

has a non-terminating repeating decimal expansion.

(iv)  $\frac{15}{1600} = \frac{15}{2^6 \times 5^2} = \frac{3 \times 5}{2^6 \times 5^2} = \frac{3}{2^6 \times 5^1}$

Hence,  $q = 2^6 \times 5^2$ , which is of the form  $2^n \times 5^m$  ( $n = 6, m = 1$ ).

So, the rational number  $\frac{15}{1600}$  has a

terminating decimal expansion.

(v)  $\frac{29}{343} = \frac{29}{7^3}$

Hence,  $q = 7^3$ , which is not of the form  $2^m \times 5^n$ . So, rational number  $\frac{29}{7^3}$  has a

non-terminating repeating decimal expansion.

(vi)  $\frac{23}{2^3 \times 5^2}$

Hence,  $q = 2^3 \times 5^2$ ,  
which is of the form  $2^n \times 5^m$  ( $n = 3, m = 2$ ).  
So, the rational number has a terminating decimal expansion.

(vii)  $\frac{129}{2^2 \times 5^7 \times 7^5}$

Hence,  $q = 2^2 \times 5^7 \times 7^5$ , which is not of the form  $2^m \times 5^n$ . So, rational number  $\frac{129}{2^2 \times 5^7 \times 7^5}$  has a non-terminating repeating decimal expansion.

(viii)  $\frac{6}{15} = \frac{2 \times 3}{3 \times 5} = \frac{2}{5}$

Hence,  $q = 5$ , which is of the form  $2^m \times 5^n$  ( $m = 0, n = 1$ ). So, the rational no  $\frac{2}{5}$  has a terminating decimal expansion.

(ix)  $\frac{35}{50} = \frac{5 \times 7}{2 \times 5^2} = \frac{7}{2 \times 5}$

Hence,  $q = 2^1 \times 5^1$ , which is of the form  $2^m \times 5^n$  ( $m = 1, n = 1$ ). So, the rational number  $\frac{35}{50}$  has a terminating decimal expansion.

(x)  $\frac{77}{210} = \frac{7 \times 11}{2 \times 3 \times 5 \times 7} = \frac{11}{2 \times 3 \times 5}$

Hence,  $q = 2 \times 3 \times 5$ , which is not of the form  $2^m \times 5^n$ . So, the rational number  $\frac{77}{210}$  has a non-terminating repeating decimal expansion.

2. Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Sol. (i)  $\frac{13}{3125} = \frac{13}{5^5} = \frac{13 \times 2^5}{5^5 \times 2^5}$   
 $= \frac{416}{10^5} = 0.00416$

(ii)  $\frac{17}{8} = \frac{17}{2^3}$   
 $= \frac{17 \times 5^3}{2^3 \times 5^3} = \frac{17 \times 5^3}{10^3} = \frac{2125}{10^3} = 2.125$

(iii)  $\frac{15}{1600} = \frac{15}{2^6 \times 5^2} = \frac{3 \times 5}{2^6 \times 5^2} = \frac{3}{2^6 \times 5}$   
 $= 0.009375$

(iv)  $\frac{3}{2^6 \times 5} = 0.115$

(v)  $\frac{6}{15} = \frac{2 \times 3}{3 \times 5} = \frac{2}{5} = 0.4$

(vi)  $\frac{35}{50} = \frac{5 \times 7}{2 \times 5^2} = \frac{7}{2 \times 5} = 0.7$

3. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational, or not.

If they are rational, and of the form  $\frac{p}{q}$ ,

what can you say about the prime factors of  $q$ ?

(i) 43.123456789

(ii) 0.120120012000120000....

(iii)  $\overline{43.123456789}$

**Sol.** (i) 43.123456789

Since, the decimal expansion terminates, so the given real number is rational and therefore of the form .

$$\begin{aligned}
 &43.123456789 \\
 &= \frac{43123456789}{1000000000} \\
 &= \frac{43123456789}{10^9} \\
 &= \frac{43123456789}{(2 \times 5)^9} \\
 &= \frac{43123456789}{2^9 \cdot 5^9}
 \end{aligned}$$

Hence,  $q = 2^9 \cdot 5^9$

The prime factorisation of  $q$  is of the form  $2^n \cdot 5^m$ , where  $n = 9$ ,  $m = 9$ .

(ii) 0.120120012000120000....

Since, the decimal expansion is neither terminating nor non-terminating repeating, therefore, the given real number is not rational.

(iii)  $\overline{43.123456789}$

Since, the decimal expansion is non-terminating and repeating, therefore, the given real number is rational.

As the number is non-terminating so  $q$  is not of the form  $2^m \times 5^n$ .